Math 215a Homework #5, Due Friday 11/9 at 1:10 PM

1. Hatcher section 2.2 exercise 8.

2. Recall that $\mathbb{CP}^n = (\mathbb{C}^{n+1} \setminus \{0\})/\sim$, where $v \sim w$ iff $v = \lambda w$ for some $\lambda \neq 0$. Show that $\mathbb{CP}^n$ has the structure of a CW complex with one cell in each dimension $0, 2, \ldots, 2n$. *Hint:* use induction on n.

3. Hatcher section 2.2 exercise 17.

4. Hatcher section 2.2 exercises 20, 21, 22. (These should be quick.)

5. Let X and Y be compact surfaces, and let $f : X \to Y$ be a d-fold branched cover[1]. Let r denote the sum of the orders of the ramification points in X. Prove the *Riemann-Hurwitz formula*

$$\chi(X) = d \cdot \chi(Y) - r.$$

6. Find an example of spaces X, Y and maps $f, g : X \to Y$ which induce the same map $H_n(X) \to H_n(Y)$ for all n, but which induce different maps $H_n(X; G) \to H_n(Y; G)$ for some n and G. (*Hint:* take $X = \mathbb{RP}^2$ and $Y = S^2$.) Why doesn't this contradict the universal coefficient theorem?

[1] f is a *branched cover* if for every $p \in X$, there are identifications of neighborhoods U of p and V of $f(p)$ with a neighborhood of the origin in $\mathbb{C}$, identifying p and $f(p)$ with 0, such that under these identifications, $f : U \to V$ is given by $f(z) = z^n$ for some positive integer n. If $n > 1$ then p is called a *ramification point* of *order* $n-1$, and $f(p)$ is a *branch point*. The adjective 'd-fold' means that if $q \in Y$ is not a branch point then $|f^{-1}(q)| = d$. Note that over the complement of the branch points, f is a covering space.