Math 215a Homework #4, Due Monday 10/29 at 1:10 PM

0. (Don't hand this in.) Show that a short exact sequence of chain complexes induces a long exact sequence on homology. Moreover, this long exact sequence is natural. (Cf. Hatcher p. 117 and p. 127.)

1. Hatcher section 2.1 problem 17(b).

2. Hatcher section 2.1 problem 18.

3. Hatcher section 2.1 problem 27.

4. Hatcher, section 2.2, problem 2.

5. If $\sigma : \Delta_n \to X$, define $\overline{\sigma} : \Delta_n \to X$ by

$$\overline{\sigma}(t_0, \ldots, t_n) := \sigma(t_n, \ldots, t_0).$$

   Define a map $T : C_n(X) \to C_n(X)$ by $T(\sigma) := (-1)^{n(n+1)/2}\overline{\sigma}$.

   (a) Show that T is a chain map.

   (b) Show (without constructing it explicitly) that there exists a chain homotopy from T to the identity.

6. (Extra credit) Hatcher section 2.1 problem 14.