Math 215a Homework #2, Due Wednesday 10/3 at 1:10 PM

1. Hatcher, section 1.3, exercise 4.

2. Hatcher, section 1.3, exercise 12.

3. Hatcher, section 1.3, exercise 14.

4. Hatcher, section 1.3, exercise 26.

5. Let n and k be positive integers, and let $F(n)$ denote the free group on n generators. Show that if G is an index k subgroup of $F(n)$, then $G \simeq F(kn - k + 1)$.

6. Let $X := T^n = \mathbb{R}^n/\mathbb{Z}^n$. Let $\widetilde{X}$ be a path connected covering space of X. Show that $\widetilde{X}$ is homeomorphic to $T^m \times \mathbb{R}^{n-m}$ for some $m \in \{0, \ldots, n\}$.

7. Let F denote the free group on two generators a, b, and let G denote the commutator subgroup of F. Show that G is freely generated by
$$\{[a^m, b^n] \mid m, n \in \mathbb{Z} \setminus \{0\}\}.$$

8. *Extra credit* (the hard part is to figure out what this means): Let (X, x_0) be path connected, locally path connected, and semi-locally simply connected. Let $\mathcal{C}$ denote the category of covering spaces $p : \widetilde{X} \to X$; here $\widetilde{X}$ does not have a base point and is not assumed to be path connected. Let $\mathcal{P}$ denote the category of sets equipped with a right $\pi_1(X, x_0)$ action. Show that there is an equivalence of categories $\mathcal{F} : \mathcal{C} \to \mathcal{P}$.