Math 215a Homework #1, Due Friday 9/21 in class

1. Let X be a path connected topological space. Show that:

   (a) $\pi_1(X, x_0)$ is completely independent of the base point x_0 (i.e. the isomorphism $\pi_1(X, x_0) \simeq \pi_1(X, x_1)$ determined by a path from x_0 to x_1 does not depend on the choice of path), if and only if $\pi_1(X, x_0)$ is abelian.

   (b) There is a canonical bijection between the set of homotopy classes of maps from S^1 to X (without any base point conditions), and the set of conjugacy classes in $\pi_1(X, x_0)$.

2. Hatcher, §1.1, Exercise 16

3. Hatcher, §1.2, Exercise 10

4. Hatcher, §1.2, Exercise 17.

5. Hatcher, §1.A, Exercise 3.

6. Let G be a topological group. (A *topological group* is a set G with both a topology and a group structure, such that the map $G \to G$ sending $x \mapsto x^{-1}$ and the map $G \times G \to G$ sending $(x, y) \mapsto xy$ are continuous.) Show that $\pi_1(G, 1)$ is abelian.

7. *Extra for experts:* Suppose that $\gamma : S^1 \to T^2$ is injective and represents a nonzero class $(a, b) \in \pi_1(T^2) = \mathbb{Z}^2$. Show that a and b are relatively prime.