Math 215a Homework #6, Due Wednesday 11/1 at 9:10 AM

1. Hatcher section 2.2 exercises 20, 21, 22. (These should be quick.)

2. Let X and Y be compact surfaces, and let $f : X \to Y$ be a degree d branched cover[1]. Let r denote the sum of the orders of the ramification points in X. Prove the *Riemann-Hurwitz formula*

$$\chi(X) = d \cdot \chi(Y) - r.$$

   Hint: start with a triangulation of Y (you can assume this exists) for which each ramification point is a vertex.

3. Hatcher section 2.2, problem 30, parts (a), (b), and (c).

4. Find an example of spaces X, Y and maps $f, g : X \to Y$ which induce the same map $H_n(X) \to H_n(Y)$ for all n, but which induce different maps $H_n(X; G) \to H_n(Y; G)$ for some n and G. (*Hint:* take $X = \mathbb{RP}^2$ and $Y = S^2$.) Why doesn't this contradict the universal coefficient theorem?

5. Hatcher, section 3.1, problem 5.

6. Hatcher, section 3.1, problem 9.

7. Let X be a topological space. Define a map $\tau : C_*(X) \to C_*(X)$ as follows. If $\sigma : \Delta_i \to X$, then $\tau(\sigma) = (-1)^{i(i+1)/2}\overline{\sigma}$, where $\overline{\sigma} : \Delta_i \to X$ is defined by $\overline{\sigma}(t_0, \ldots, t_i) = \sigma(t_i, \ldots, t_0)$.

   (a) Show that τ is a chain map.

   (b) Use the method of acyclic models to show that there is a chain homotopy $K : C_*(X) \to C_{*+1}(X)$ with $\partial K + K\partial = \tau - \mathrm{id}_{C_*(X)}$. *Hint:* Given $\sigma : \Delta_i \to X$, define

$$K(\sigma) = \sigma_\#(K_i),$$

   where $K_i \in C_{i+1}(\Delta_i)$ is defined by induction to satisfy

$$\partial K_i + K(\partial \,\mathrm{id}_{\Delta_i}) = \tau(\mathrm{id}_{\Delta_i}) - \mathrm{id}_{\Delta_i}.$$

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[1]This means that for every $p \in X$, there are identifications of neighborhoods of p and $f(p)$ with $\mathbb{C}$ identifying p and $f(p)$ with 0, such that under these identifications, $f(z) = z^n$ near the origin where n is a positive integer. So if $n = 1$ then f is a local homeomorphism at p; while if $n > 1$ then p is called a *ramification point* of *order* $n-1$, and $f(p)$ is a *branch point*. Over the complement of the branch points, f is a covering, and d is its degree.