

Math 215a Homework #3, Due Monday 9/25/17 at 9:10 AM

1. Hatcher, section 1.3, exercise 4.
2. Hatcher, section 1.3, exercise 12.
3. Hatcher, section 1.3, exercise 14.
4. Hatcher, section 1.3, exercise 26.
5. Let n and k be positive integers, and let $F(n)$ denote the free group on n generators. Show that if G is an index k subgroup of $F(n)$, then $G \simeq F(kn - k + 1)$.
6. Let $X := T^n = \mathbb{R}^n / \mathbb{Z}^n$. Let \tilde{X} be a path connected covering space of X . Show that \tilde{X} is homeomorphic to $T^m \times \mathbb{R}^{n-m}$ for some $m \in \{0, \dots, n\}$.
7. Let F denote the free group on two generators a, b , and let G denote the commutator subgroup of F . Show that G is freely generated by

$$\{[a^m, b^n] \mid m, n \in \mathbb{Z} \setminus \{0\}\}.$$