Math 214 HW#1, due 2/5/13 at 2:10 PM

1. Lee 1.7. (Just do the case $n = 2$.)

2. Lee 1.9. (Just do the case $n = 1$.) Also check that the projection $\mathbb{C}^2 \setminus \{0\} \to \mathbb{C}P^1$ is smooth.

3. Show that $\mathbb{C}P^1$ is diffeomorphic to S^2.

4. Consider spherical coordinates on $\mathbb{R}^3$ (not including the line $z = 0$) ρ, ϕ, θ defined in terms of the Euclidean coordinates x, y, z by

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi.$$

   (a) Express $\partial/\partial\rho$, $\partial/\partial\phi$, and $\partial/\partial\theta$ as linear combinations of $\partial/\partial x$, $\partial/\partial y$, and $\partial/\partial z$. (The coefficients in these linear combinations will be functions on $\mathbb{R}^3 \setminus (z = 0)$.)

   (b) Express $d\rho$, $d\phi$, and $d\theta$ as linear combinations of dx, dy, and dz.

5. Let V and W be finite dimensional vector spaces and let $A : V \to W$ be a linear map. Show that the dual map $A^* : W^* \to V^*$ is given in coordinates as follows. Let $\{e_i\}$ and $\{f_j\}$ be bases for V and W, and let $\{e^i\}$ and $\{f^j\}$ be the corresponding dual bases for V^* and W^*. If $Ae_i = A_i^j f_j$ then $A^* f^j = A_i^j e^i$.

6. Let V be a finite dimensional vector space and let $\langle \cdot, \cdot \rangle$ be an inner product on V. The inner product determines an isomorphism $\phi : V \to V^*$.

   (a) Show that the isomorphism ϕ is given in coordinates as follows. Let $\{e_i\}$ be a basis for V, let $\{e^i\}$ be the dual basis, and write $g_{ij} = \langle e_i, e_j \rangle$. Then $\phi(e_i) = g_{ij} e^j$.

   (b) The inner product, together with the isomorphism ϕ, define an inner product on V^*. Write this in coordinates as $g^{ij} = \langle e^i, e^j \rangle$. Show that the matrix (g^{ij}) is the inverse of the matrix (g_{ij}).

7. Show that if M and N are smooth manifolds and $p \in M$, $q \in N$, then there is a canonical isomorphism $T_{(p,q)}(M \times N) = T_pM \oplus T_qN$. Describe this isomorphism in terms of derivations, coordinate charts, and velocity vectors of curves.

8. How difficult was this assignment?