1. This is a multiple choice question. Which of the following is within \( \frac{1}{2} \) of \( \int_0^1 xe^x \, dx \).

(a) 1  
(b) \( e \)  
(c) \( e - 2 \)  
(d) \( \frac{3}{4} \)

2. This is a multiple choice question. If \( f(x) > 0 \) for all \( x > 0 \) and \( \int_0^\infty f(x) \, dx \) diverges, then

(a) \( \lim_{x \to \infty} f(x) \neq 0 \).  
(b) \( f(x) > x \) for all \( x > 0 \).  
(c) \( \sum_{n=0}^\infty f(n) \) diverges.  
(d) If one makes a solid of revolution around \( x \)-axis, then the volume is infinite.

3. Evaluate the integral 
\[
\int x^3 \ln(1 + x^2) \, dx.
\]

4. Evaluate the integral if it converges, or show that it is divergent 
\[
\int_0^1 \frac{dx}{x(x+1)^2}.
\]

5. This is a multiple choice question. If \( \sum a_n \) converges absolutely, then

(a) \( \sum_n |a_n|^{\frac{1}{2}} \) converges.
(b) \( \sum_{n} (-1)^n |a_n|^\frac{1}{2} \) converges.

(c) \( \sum_{n} a_n^2 \) converges.

(d) \( \sum_{n} \sin(a_n) \) converges.

6. This is a multiple choice question. The 4th Taylor polynomial \( T_4(x) \) of \( \frac{e^x}{1+x} \) expanded about \( a = 0 \) is:

(a) \( 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} \).

(b) \( 1 - \frac{x^2}{2} - \frac{x^3}{3} + \frac{3x^4}{8} \).

(c) \( 1 + 2x + \frac{3}{2}x^2 + \frac{2}{3}x^3 + \frac{5}{24}x^4 \).

(d) \( 1 + x + \frac{x^2}{4} + \frac{3x^3}{5} + \frac{23}{43}x^4 \).

(e) none of the above.

7. This is a “true-false” question. If true, explain why. If false, give a counter-example.

(a) If \( a_n \neq 0 \) and \( a_{n+1}/a_n < 1 \), then \( \lim_{n \to \infty} a_n = 0 \).

(b) If \( \{a_n\} \) and \( \{b_n\} \) are divergent, then \( \{a_n + b_n\} \) is divergent.

8. Find the radius and the interval of convergence of the power series

\[ \sum_{n=2}^{\infty} \frac{1}{\ln n} (x - 2)^n. \]

9. Find general solution to the differential equation

\[ x(x+1)y' + y = x. \]
10. Solve the initial value problem

\[ y'' - 4y = 2e^{2x}, \quad y(0) = 0, \quad y'(0) = ???. \]

11. Find general solution in the form of power series for the initial problem

\[ xy'' - xy' - y = 0, \quad y(0) = 0, \quad y'(0) = 1. \]

12. Find general solution to the differential equation

\[ y' = (1 + y)^3 \ln(1 + x). \]

13. Find general solution to the differential equation

\[ y'' - y = \frac{2}{e^x - e^{-x}}. \]