1. 10 pts Evaluate the integral
\[ \int e^{\sqrt{x}} \frac{dx}{\sqrt{x}} \]
2.15 \textit{pts} Evaluate the integral

$$\int \frac{1}{(t+1)^2(t-1)} \, dx$$
3.15 pts Indicate which of the following statements are true and which are false. Do not show your work.

1. \( \int_1^\infty \frac{1 - e^{-x}}{x^3} \, dx \) converges by comparison test with \( \int_1^\infty \frac{1}{x^3} \, dx \).

2. \( \int_0^1 \frac{\sin(x)}{x} \, dx \) diverges by comparison test with \( \int_0^1 \frac{1}{x} \, dx \).

3. \( \int_0^1 \frac{dx}{\sqrt{x^2 + x}} \) is a convergent improper integral.

4. \( \int_1^\infty \frac{1}{(x - 1)^2} \, dx \) is a divergent improper integral.

5. \( \int_0^1 \frac{\ln(x)}{x^{1/2}} \, dx \) is a convergent improper integral.
4.15 pnts Find the radius and the interval of convergence of the power series

\[
\sum_{n=1}^{\infty} \frac{n - 1}{(n + 2)(2n + 5)} \left( \frac{x}{2} \right)^n
\]
5.15 pts State whether the following series is absolutely convergent, conditionally convergent, or divergent. Do not show your work.

1. \[ \sum_{n=1}^{\infty} \cos\left(\frac{\pi}{n^2}\right) \]

2. \[ \sum_{n=1}^{\infty} (-1)^n \frac{13 + n + e^{-n}}{13 + 4n^2} \]

3. \[ \sum_{n=1}^{\infty} \left( \frac{1}{\arctan n + 1} - \frac{1}{\arctan n} \right) \]

4. \[ \sum_{n=1}^{\infty} \frac{3^n}{n^3} (-1)^n \]

5. \[ \sum_{n=2}^{\infty} \cos(\pi n) \frac{1}{n \ln^2(n)} \]
6.15 points For each statement indicate whether it is true or false. Do not show your work.

1. If \( \sum_{n=1}^{\infty} c_n \) converges, then \( \sum_{n=1}^{\infty} (-1)^n c_n^2 \) also converges.

2. If \( f(x) < 0 \) is continuous and \( \int_{1000}^{\infty} f(x) \, dx \) is convergent then \( \sum_{n=1}^{\infty} f(n) \) converges.

3. If the sequence \( \{a_n\} \) diverges and the series \( \sum_{n=0}^{\infty} b_n \) diverges then \( \sum_{n=0}^{\infty} a_n b_n \) diverges.

4. If the sequence \( \{a_n\} \) diverges and and the sequence \( \{b_n\} \) diverges then \( \{a_n b_n\} \) diverges.

5. If \( \sum_{n=0}^{\infty} a_n (x - 2)^n \) converges for \( x = 7 \) and \( \sum_{n=0}^{\infty} a_n (-6)^n \) diverges, then \( \sum_{n=0}^{\infty} a_n (x - 2)^n \) diverges for \( x = -6 \).
7.15 pts For each statement indicate whether it is true or false. Do not show your work.

1. \( \sum_{n=1}^{\infty} nc_n x^n \) converges absolutely inside (excluding endpoints) of the interval of convergence of the power series \( \sum_{n=1}^{\infty} c_n x^n \).

2. \( \sum_{n=1}^{\infty} c_n x^n \) has radius of convergence \( R \), then \( \sum_{n=1}^{\infty} c_n R^n \) converges conditionally.

3. \( \sum_{n=1}^{\infty} c_n x^n \) diverges for \( |x| > R \), then \( R \) is the radius of convergence of this power series.

4. \( \sum_{n=1}^{\infty} c_n x^n \) converges for \( 0 < |x| < a \), then \( a < R \) where \( R \) is the radius of convergence of this power series.

5. The radius of convergence of \( \sum_{n=1}^{\infty} \frac{x^n}{n} + \sum_{n=1}^{\infty} \frac{(2x)^n}{n^{100}} \) is 1.
8.15 pts Solve the initial-value problem.

\[ e^x (-2y y' + 2y') = 1, \quad y(0) = 2. \]
9.15 pts Find the general solution to the differential equation

\[ \frac{dy}{dx} = x + \ln(x) + xy + \ln(x)y, \]
10.10 pts Find the general solution to the differential equation

\[ y'' + y = \cos x, \quad y(0) = 0, \quad y'(0) = \frac{5}{2}. \]
11.15 pnts Match pictures to differential equations.

1. \( \frac{dy}{dx} = y^2 \)

2. \( \frac{dy}{dx} = y \)

3. \( \frac{dy}{dx} = y^{1/2} \)

4. \( \frac{dy}{dx} = y^{-2} \)

5. \( \frac{dy}{dx} = y^{-1} \)
12. 20 pts Find the power series solution to the differential equation:

\[ y'' - xy' - y = 0, \quad y(0) = 1, \quad y'(0) = 0. \]