Math 185 HW#3, due 2/16/16 at 8:10 AM

0. (optional, not to be graded, but might be useful practice) Gamelin page 57, exercise 1, and page 67, exercise 1.

1. Gamelin, page 58, exercises 6, 7. (You may assume the result of page 57, exercise 5.)

2. Gamelin, page 62, exercises 4, 5.

3. Gamelin, page 68, exercises 2, 4.

4. Find a conformal bijection from the open first quadrant ($x > 0, y > 0$) to the unit disk ($|z| < 1$). *Hint:* first map to the upper half plane, then use an appropriate linear fractional transformation.

5. Given distinct points $z_1, z_2, z_3, z_4 \in \widehat{\mathbb{C}}$, define the *cross ratio*

$$(z_1, z_2, z_3, z_4) = \frac{(z_1 - z_3)(z_2 - z_4)}{(z_1 - z_4)(z_2 - z_3)}.$$

   (If one of the points z_k is ∞, cross out the two factors containing z_k.)

   (a) Show that the cross ratio is $f(z_1)$, where f is the unique linear fractional transformation sending $z_2 \mapsto 1$, $z_3 \mapsto 0$, and $z_4 \mapsto \infty$.

   (b) Given another four distinct points $w_1, w_2, w_3, w_4 \in \widehat{\mathbb{C}}$, show that there exists a linear fractional transformation sending $z_k \mapsto w_k$ for $k = 1, \ldots, 4$ if and only if $(z_1, z_2, z_3, z_4) = (w_1, w_2, w_3, w_4)$.

   (c) Show that (z_1, z_2, z_3, z_4) is real if and only if the four points z_1, z_2, z_3, z_4 lie on a line or a circle.