

Math 185 HW#4, due 9/25/12 at 12:40 PM

1. Gamelin, page 68, exercises 2, 4.
2. Find a conformal bijection from the open first quadrant ($x > 0, y > 0$) to the unit disk ($|z| < 1$). *Hint*: first map to the upper half plane, then use an appropriate linear fractional transformation.

3. Given distinct points $z_1, z_2, z_3, z_4 \in \widehat{\mathbb{C}}$, define the *cross ratio*

$$(z_1, z_2, z_3, z_4) = \frac{(z_1 - z_3)(z_2 - z_4)}{(z_1 - z_4)(z_2 - z_3)}.$$

(If one of the points z_k is ∞ , cross out the two factors containing z_k .)

- (a) Show that the cross ratio is $f(z_1)$, where f is the unique linear fractional transformation sending $z_2 \mapsto 1$, $z_3 \mapsto 0$, and $z_4 \mapsto \infty$.
 - (b) Given another four distinct points $w_1, w_2, w_3, w_4 \in \widehat{\mathbb{C}}$, show that there exists a linear fractional transformation sending $z_k \mapsto w_k$ for $k = 1, \dots, 4$ if and only if $(z_1, z_2, z_3, z_4) = (w_1, w_2, w_3, w_4)$.
 - (c) Show that (z_1, z_2, z_3, z_4) is real if and only if the four points z_1, z_2, z_3, z_4 lie on a line or a circle.
4. Gamelin, page 75, exercises 3, 4.
 5. Gamelin, page 82, exercises 1, 2, 3.