Math 113 Homework # 5, due 3/6/03 at 8:10 AM

1. Fraleigh section 8, problem 21; section 9, problem 23.

2. Show that if one performs 8 perfect shuffles of a deck of cards, then this returns the cards to their original position. Note that a perfect shuffle can be represented by the following permutation $f \in S_{52}$:

$$f(x) = \begin{cases} 2x-1, & \text{if } x \in \{1,\ldots,26\}, \\ 2(x-26), & \text{if } x \in \{27,\ldots,52\}. \end{cases}$$

   Suggestion: first express f as a product of disjoint cycles.

3. (a) Show that if $\mu = (x_1\ x_2\ \cdots\ x_k) \in S_n$ is a k-cycle and $\sigma \in S_n$ is any permutation then $\sigma\mu\sigma^{-1}$ is the k-cycle

$$\sigma\mu\sigma^{-1} = (\sigma(x_1)\ \sigma(x_2)\ \cdots\ \sigma(x_k)).$$

   (b) Using the above, can you guess a necessary and sufficient condition for two permutations in S_n to be conjugate to each other?

4. (a) Find the left and right cosets of the subgroup $H = \{R_0, F_0\}$ of D_4. Are they the same?

   (b) Same question for $H = \{R_0, R_2\}$.

D_4	R_0	R_1	R_2	R_3	F_0	F_1	F_2	F_3
R_0	R_0	R_1	R_2	R_3	F_0	F_1	F_2	F_3
R_1	R_1	R_2	R_3	R_0	F_1	F_2	F_3	F_0
R_2	R_2	R_3	R_0	R_1	F_2	F_3	F_0	F_1
R_3	R_3	R_0	R_1	R_2	F_3	F_0	F_1	F_2
F_0	F_0	F_3	F_2	F_1	R_0	R_3	R_2	R_1
F_1	F_1	F_0	F_3	F_2	R_1	R_0	R_3	R_2
F_2	F_2	F_1	F_0	F_3	R_2	R_1	R_0	R_3
F_3	F_3	F_2	F_1	F_0	R_3	R_2	R_1	R_0

5. Fraleigh section 10 exercises 34, 39, 40, 44.

6. Extra credit: Show that every even permutation in A_n can be expressed as a product of 3-cycles. (The 3-cycles need not be disjoint. Also, we regard the identity as the product of zero 3-cycles.) Suggestion: use induction on n.

7. How challenging did you find this assignment? How long did it take?