Math 113 Homework # 1, due 1/30/03 at 8:00 AM

This is kind of a warm-up assignment because we haven't introduced much material yet in the first two lectures.

1. Fraleigh, section 0, exercise 12 and exercises 29–34.

2. (a) Let $f : X \to Y$ and $g : Y \to Z$. Show that if $g \circ f : X \to Z$ is surjective, then g is surjective. Show that if $g \circ f$ is injective, then f is injective.

   (b) Show that $f : X \to Y$ is bijective if and only if there exists $g : Y \to X$ with $g \circ f = \mathrm{id}_X$ and $f \circ g = \mathrm{id}_Y$.

   (c) Show that if X and Y are finite sets with the same cardinality and $f : X \to Y$, then f is injective if and only if f is surjective.

3. Fix a positive integer n. Let us try to define multiplication mod n by $[x][y] = [xy]$. Is this well-defined? Why or why not?

4. (a) Prove by induction on n that the sum of the first n odd positive integers is n^2:
$$1 + 3 + \cdots + (2n - 1) = n^2.$$

   (b) Show by induction on n that if n is a nonnegative integer and x is a real number with $x \neq 1$ then
$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}.$$

   Just for fun (not required), can you see the above formulas without using induction?

5. Show that for any positive integer n, a $2^n \times 2^n$ checkerboard with one square removed can be tiled by L-triominoes. (An "L-triomino" is a shape consisting of three squares joined in an 'L'-shape. To "tile" means to completely cover, using tiles that do not overlap.)

6. How challenging did you find this assignment? How long did it take?