Qualifying Exam Syllabus

Holly Mandel

10:00 AM - 1:00 PM, April 26, 2019

Committee: David Nadler (Exam Chair), Song Sun (Advisor), Michael Christ, Hitoshi Murayama (Academic Senate Representative).

1 Major topic: Complex Geometry

Reference: Principles of Algebraic Geometry, Griffiths and Harris

- Local theory, sheaf, Dolbeault, and de Rham cohomology
- Intersection product, Poincaré duality
- Holomorphic vector bundles, Hermitian metrics, connections, curvature
- Hodge theorem, Kodaira-Serre duality
- Kähler manifolds, Hodge identities, Hodge decomposition, Lefschetz decomposition
- Divisors and line bundles, linear systems, Chern classes of line bundles
- Kodaira vanishing theorem, Lefschetz hyperplane theorem, Lefschetz theorem on (1,1)-classes
- Chow's theorem, degree of a variety, tangent spaces to algebraic varities
- Kodaira embedding theorem, blowing up
- Projective embedding of Riemann surfaces, Riemann-Hurwitz formula, degree-genus formula, Riemann-Roch theorem

2 Major topic: Riemannian Geometry

Reference: Riemannian Geometry, Do Carmo

- Connections and covariant differentiation
- Curvature, sectional curvature, Ricci curvature, scalar curvature, symmetries, geometric interpretations
- Geodesics, the exponential map, convex neighborhoods, cut locus, injectivity radius
- Jacobi fields, conjugate points
- Isometric immersions, Gauss-Codazzi equations
- Completeness, Hopf-Rinow theorem, Hadamard theorem

- Spaces of constant curvature
- Variations of energy, Bonnet-Myers and Synge-Weinstein theorems
- Morse index theorem

3 Minor topic: Harmonic Analysis

Reference: Classical Fourier Analysis, Grafakos

- Fourier Transform on abelian groups, torus, $\mathbb{R}^d,$ Parseval's Theorem, Plancherel's theorem, convolution
- Schwartz space, tempered distributions
- Decay of Fourier coefficients
- Uniform and pointwise convergence, Kolmogorov's theorem, L^p convergence
- Wiener's Tauberian theorem
- Riesz-Thorin theorem
- Weak L^p , Hardy-Littlewood maximal function, Lebesgue differentiation theorem
- Calderón-Zygmund decomposition
- BMO, John-Nirenberg inequality
- Oscillation bounds and differentiation
- Singular integral operators: L^p boundedness, homogeneous distributions, almost-everywhere existence of principal value integrals, almost-everywhere differentiability