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Isomorphisms of differential forms and cochains

Abstract

This paper proves an isomorphism theorem for cochains and differential forms, before passing to cohomology. De Rham's theorem is a consequence. This leads to an extension of much of calculus and homology theory to nonsmooth domains, called chainlets,*and makes available combinatorial techniques for smooth domains that limit to the classic analytic methods. We find maximal subspaces of L^1 forms that satisfy Stokes's theorem for domains of chainlets giving a measurable, as well as optimal, extension of the theory.*

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Isomorphisms of Differential Forms and Cochains

By *Jenny Harrison*

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1. Introduction

A differential form acts as a linear functional on the vector space of simplicial chains via integration and is therefore a cochain. De Rham's theorem tells us the integration mapping Ψ sending L^1 differential forms into cochains

$$\Psi(\omega) \cdot A = \int_A \omega$$

induces an isomorphism of cohomology rings. This paper is a study of the mapping Ψ. Theorem 2.12 shows that Ψ is an isomorphism at the level of smooth differential forms and cochains before passing to cohomology. Since Ψ commutes with the exterior derivative on forms and the codifferential operator on cochains, de Rham's theorem is a direct corollary. Poincaré duality, which relies on de Rham's theorem, extends to chains and forms [5].

In the 1950s Whitney [8] and Wolfe [9] identified spaces of cochains isomorphic to Lipschitz and flat differential forms, respectively. These isomorphisms allow one to move between the infinitesimal and the global, providing a link between analysis and geometry. The link is tenuous, however, since basic operators on differential forms such as the exterior derivative, Hodge star, and pullback are not all continuous nor even defined in these spaces.

The duality between "smoothness" of mappings and "roughness" of invariant sets appearing in the C^2 Seifert conjecture counterexamples[1] of [2] suggested the isomorphisms presented here. Our isomorphisms *do* preserve the basic operators, enriching the links between topology, measure theory, dynamical systems, fractal geometry, and physics. A common language between these subjects emerges with common operators and cross-fertilization of results.

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[1] This duality is not present in the C^∞ Seifert counterexamples of Kuperberg [6].

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