Perturbative Gadgets

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summary

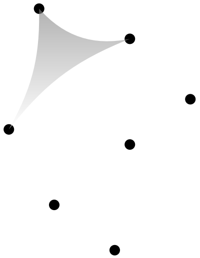
perturbative gadgets following Jordan and Farhi, arxiv 0802.1874

- background and idea of problem
- simplified setting
- perturbation theory: general
- perturbation theory: simplified setting
- numerical simulation
- commentary

some other maths

- geometry of AdS/CFT
- spectral theory
- geodesics
- error correcting codes

background and idea of problem



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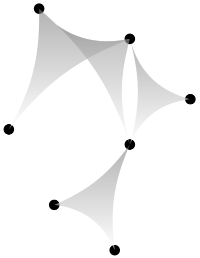

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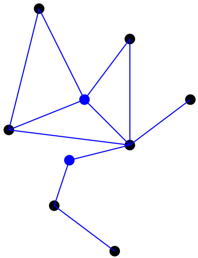

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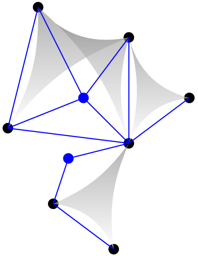

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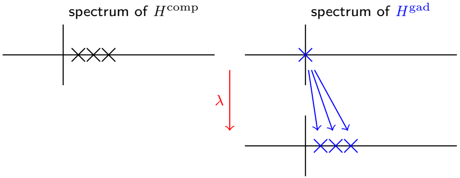

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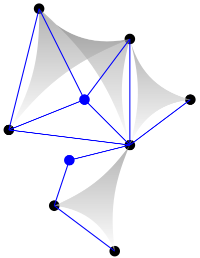

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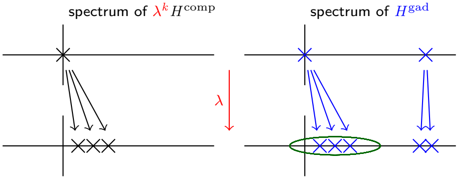

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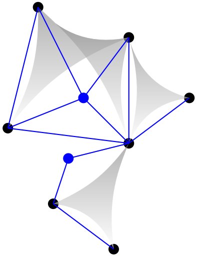

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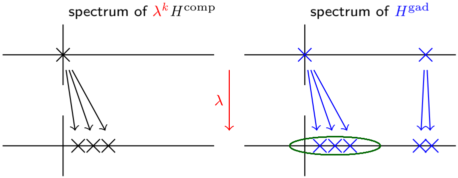

$$\text{low.energy.sec}(H^{\text{gad}}) \sim \lambda^k H^{\text{comp}} + \mathcal{O}(\lambda^{k+1})$$

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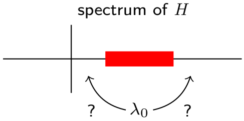


- k-local Hamiltonian is QMA complete.

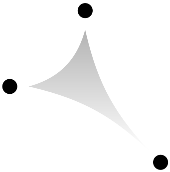
  Kempe**-Kitaev*-Regev** $k = 2$, 2005
  * $k = 5$ in 2002, ** $k = 3$ in 2003,

- adiabatic computation = circuit model

  Aharonov et al $k = 3$, 2004
  KKR $k = 2$

Oliveira-Terhal 2005,
Jordan-Farhi 2008,
Cao-Kais 2016,
Cubitt-Montanaro-Piddock 2017

simplified setting



$$H^{\text{comp}} := \sigma_1\sigma_2\sigma_3$$

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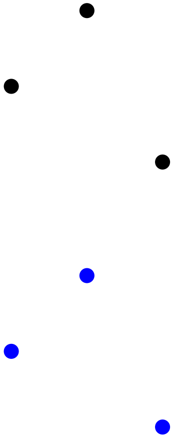

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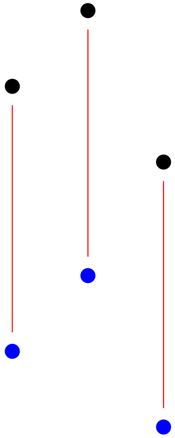

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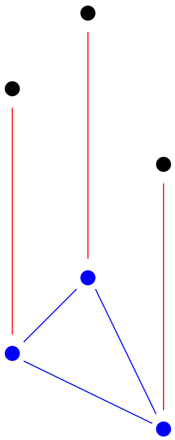

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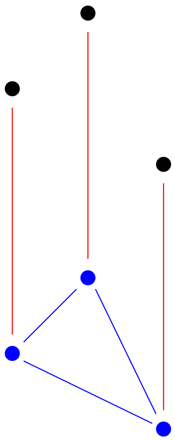

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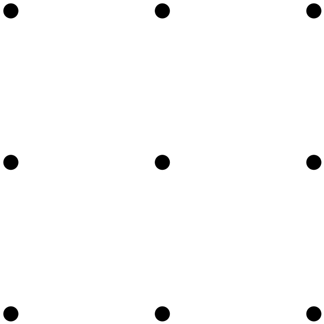
plaquette geometry



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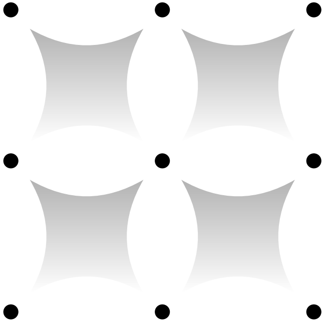

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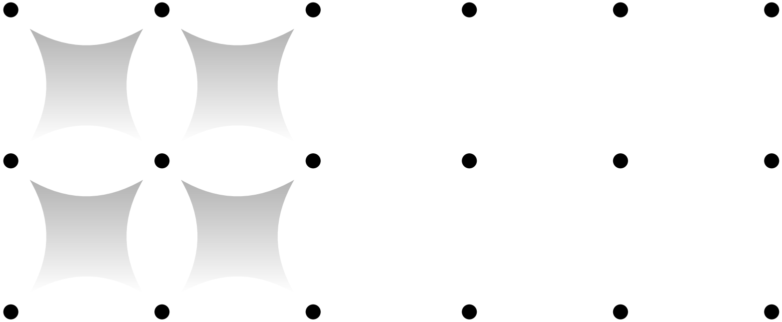

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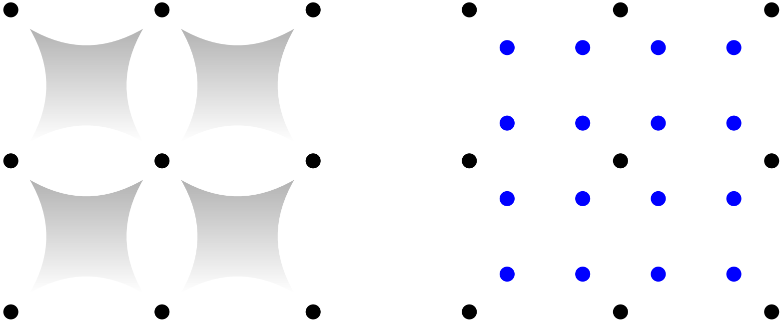

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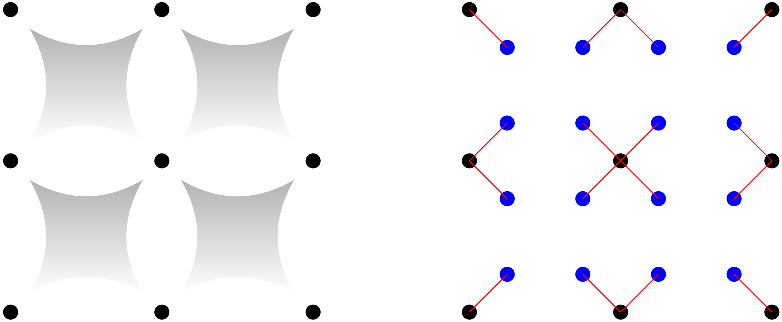

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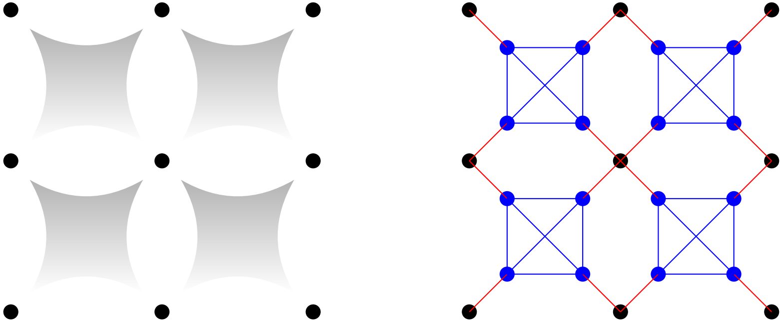

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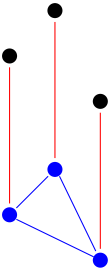

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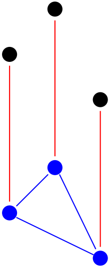

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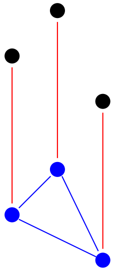

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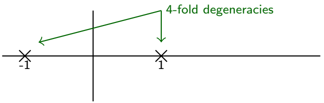
spectrum of H^{comp}



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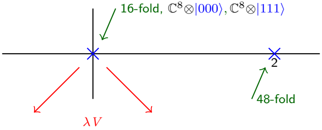

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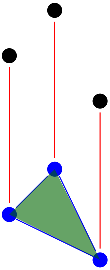

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$$X_1 X_2 X_3$$

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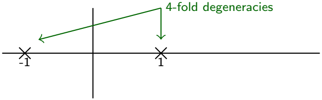

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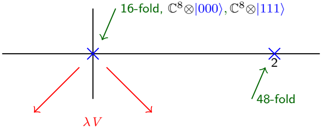

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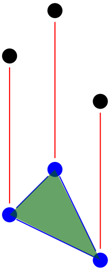

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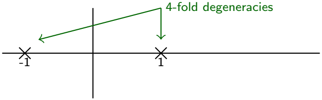

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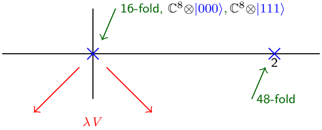

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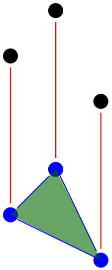

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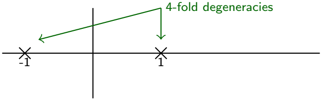

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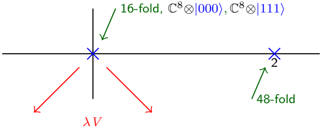

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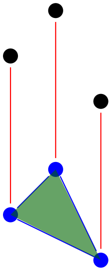

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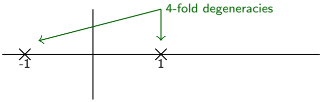

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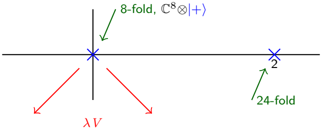

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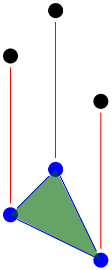

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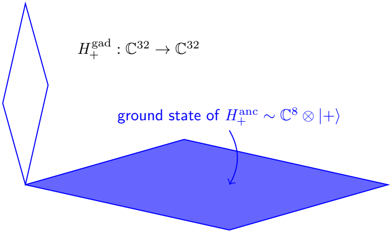
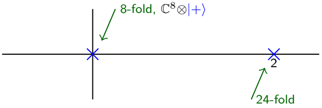


spectrum of H_+^{anc}



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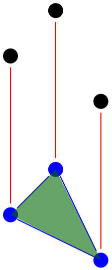

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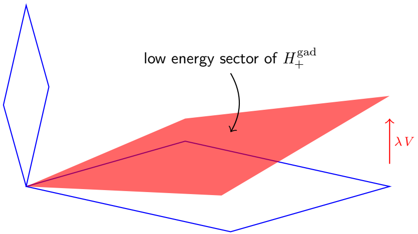

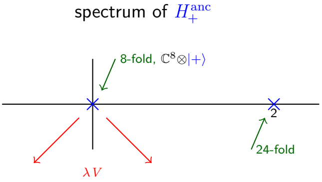

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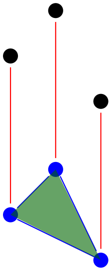

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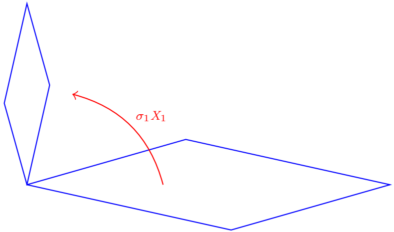
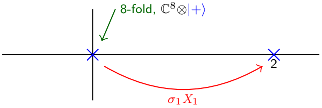


spectrum of H^{anc}_{+}



perturbation theory: general

$$H : \mathbb{C}^n \to \mathbb{C}^n \qquad H := H_0 + \hat{V} \qquad \hat{V} := \lambda V \qquad \ker H_0 =: \mathcal{E}_0 \qquad \dim \mathcal{E}_0 =: d$$

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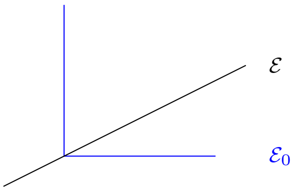

$$\mathcal{E} := \mathrm{span}\{|\psi_i\rangle\}_{i=1}^{d}$$

where $|\psi_i\rangle$ are d^{th} lowest eigenvectors of H with energies E_i

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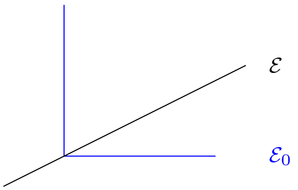


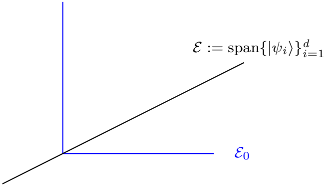
$$\mathcal{E} := \mathrm{span}\{|\psi_i\rangle\}_{i=1}^{d}$$

where $|\psi_i\rangle$ are d^{th} lowest eigenvectors of H with energies E_i
define effective hamiltonian which captures low energy sector $\mathcal{E}$

$$H_{\text{eff}}(H, d) := \sum_{i=1}^{d} E_i |\psi_i\rangle\langle\psi_i|$$

perturbation theory: general

$$H : \mathbb{C}^n \to \mathbb{C}^n \qquad H := H_0 + \hat{V} \qquad \hat{V} := \lambda V \qquad \ker H_0 =: \mathcal{E}_0 \qquad \dim \mathcal{E}_0 =: d$$



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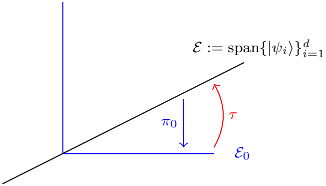

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need to study the operator $A := \pi_0 \hat{V} \tau : \mathcal{E}_0 \to \mathcal{E}_0$ since

$$H_{\text{eff}}(H, d) = \tau A \tau^\dagger$$

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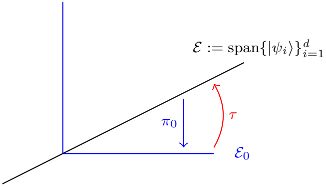

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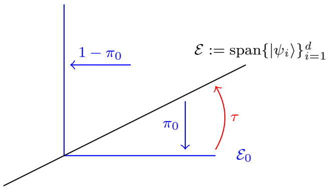

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$$\tau := \sum_{k=0}^{\infty} \lambda^k \tau^{(k)} \qquad A := \sum_{k=1}^{\infty} \lambda^k A^{(k)}$$

$$S^0 := -\pi_0$$
$$S^k := (-1)^k H_0{}^{-k}(1 - \pi_0)$$

perturbation theory: general

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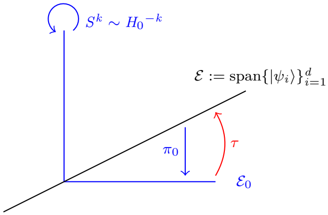

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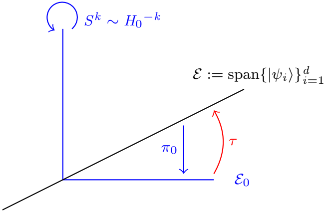

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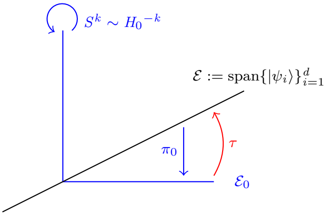

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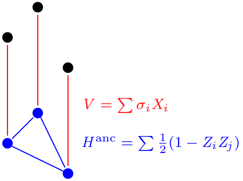
$p_i \geq 0$ for all p_i

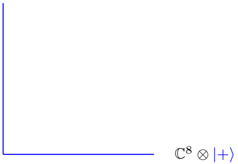
$p_1 + \cdots + p_{k'} \geq k'$ for all $k' \leq k$

$p_1 + \cdots + p_k = k$

perturbation theory: simplified setting



$$H_{\mathrm{eff}}(H_+^{\mathrm{gad}}, 8) = \tau A \tau^\dagger$$



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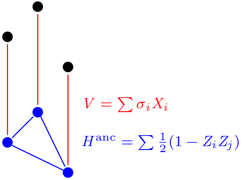
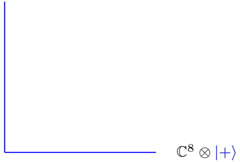
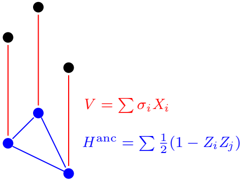


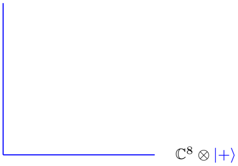
$$H_{\text{eff}}(H_+^{\text{gad}}, 8) = \tau A \tau^\dagger$$

$$A = \quad \lambda A^{(1)} + \lambda^2 A^{(2)} + \lambda^3 A^{(3)} + \mathcal{O}(\lambda^4)$$



perturbation theory: simplified setting





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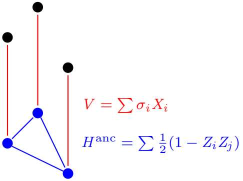

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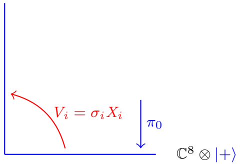
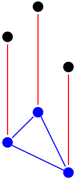
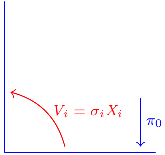


perturbation theory: simplified setting





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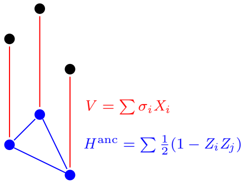
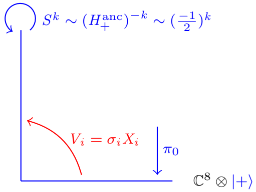
$$= \lambda(0)$$

perturbation theory: simplified setting





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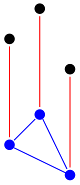
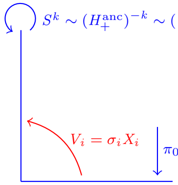
perturbation theory: simplified setting





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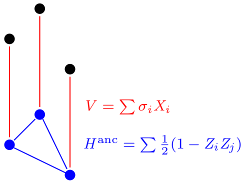
$$= \lambda(0) + \lambda^2 (\tfrac{-3}{2} \pi_0)$$

perturbation theory: simplified setting



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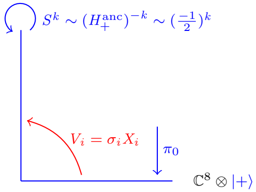
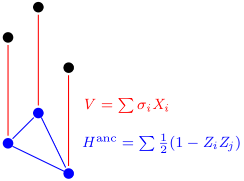


$$= \lambda(0) + \lambda^2(\frac{-3}{2}\pi_0) + \lambda^3(\frac{3}{2}\pi_0 \sigma_1 \sigma_2 \sigma_3 \pi_0) + \mathcal{O}(\lambda^4)$$

perturbation theory: simplified setting



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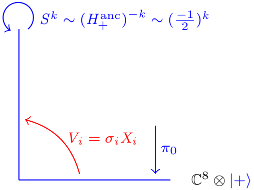
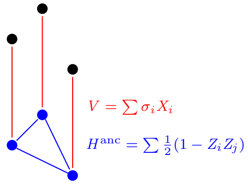


$$H_{\text{eff}}(H_+^{\text{gad}}, 8) = \tfrac{-3}{2}\lambda^2 \pi_{\text{low energy}} + \tfrac{3}{2}\lambda^3 H^{\text{comp}} \otimes \pi_0 + \mathcal{O}(\lambda^4)$$

perturbation theory: simplified setting



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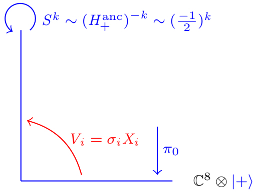
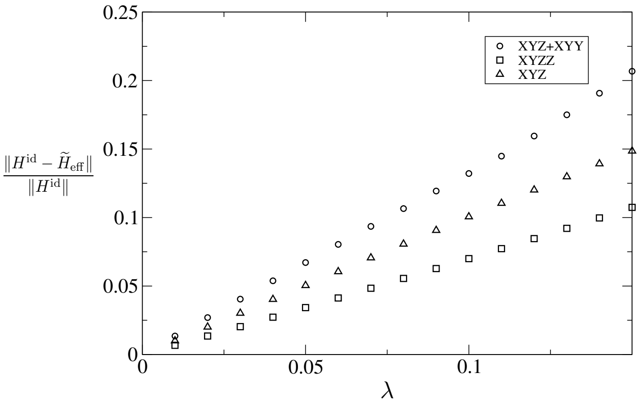


$$= \lambda(0) + \lambda^2 (\tfrac{-3}{2} \pi_0) + \lambda^3 (\tfrac{3}{2} \pi_0 \sigma_1 \sigma_2 \sigma_3 \pi_0) + \mathcal{O}(\lambda^4)$$

$$\tilde{H}_{\text{eff}}(H_+^{\text{gad}}, 8, \Delta) = \tfrac{3}{2} \lambda^3 H^{\text{comp}} \otimes \pi_0 + \mathcal{O}(\lambda^4)$$

numerical simulation



$$\tilde{H}_{\mathrm{eff}}(H_{+}^{\mathrm{gad}}, 2^k, \Delta) = c\lambda^k H^{\mathrm{comp}} \otimes \pi_0 + \mathcal{O}(\lambda^{k+1})$$

commentary

- spectral shift

$$H_{\text{eff}}(H_{+}^{\text{gad}}, 8) = \frac{-3}{2}\lambda^2 \pi_{\text{low energy}} + \frac{3}{2}\lambda^3 H^{\text{comp}} \otimes \pi_0 + \mathcal{O}(\lambda^4)$$

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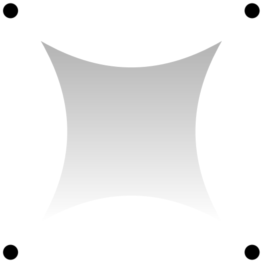
- varying-body interactions



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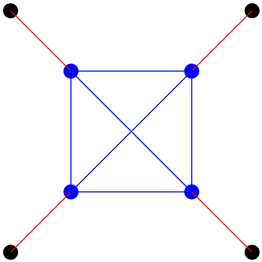

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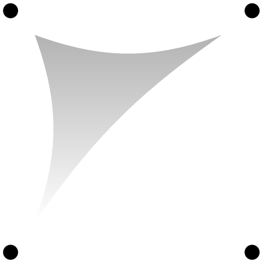

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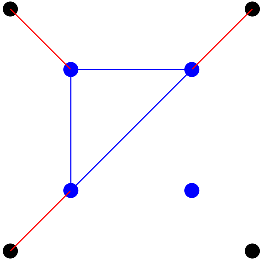

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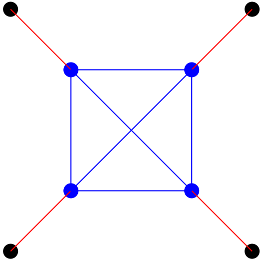
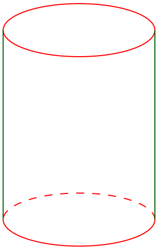
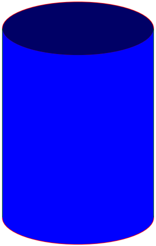


geometry of AdS/CFT



boundary $\sim \mathbb{R} \times \mathbb{S}^{d-1}$

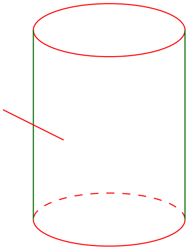
geometry of AdS/CFT



boundary $\sim \mathbb{R} \times \mathbb{S}^{d-1}$

bulk $\sim \mathrm{AdS}_{d+1}$

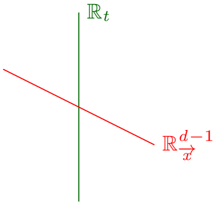
geometry of AdS/CFT



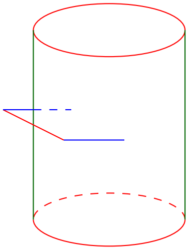
boundary $\sim \mathbb{R} \times \mathbb{S}^{d-1}$

boundary metric $\sim -dt^2 + dx^2$

bulk $\sim \mathrm{AdS}_{d+1}$



geometry of AdS/CFT

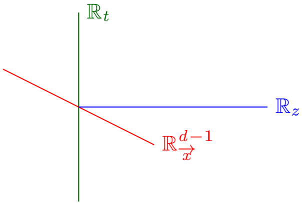


$$\text{boundary} \sim \mathbb{R} \times \mathbb{S}^{d-1}$$

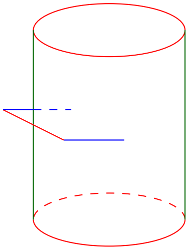
$$\text{boundary metric} \sim -dt^2 + dx^2$$

$$\text{bulk} \sim \text{AdS}_{d+1}$$

$$\text{bulk metric} \sim \frac{dz^2 - dt^2 + dx^2}{z^2}$$



geometry of AdS/CFT



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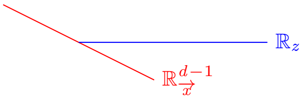
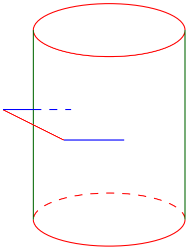


constant time slice (or wick rotation) gives hyperbolic geometry

Poincaré metric $\sim \dfrac{dz^2 + dx^2}{z^2}$

geometry of AdS/CFT



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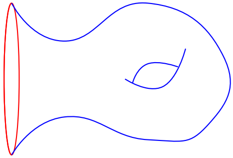

constant time slice (or wick rotation) gives hyperbolic geometry

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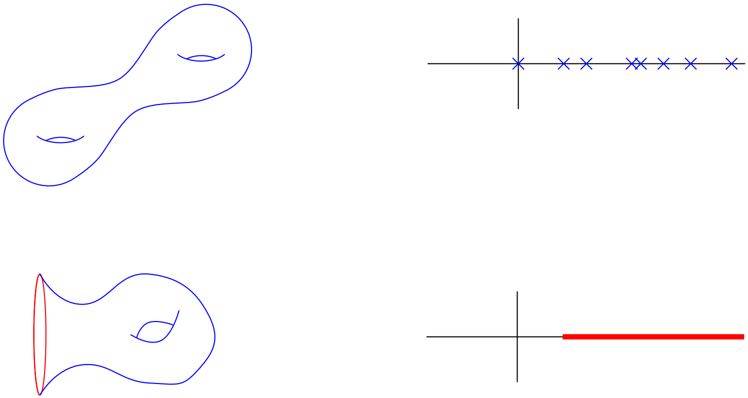
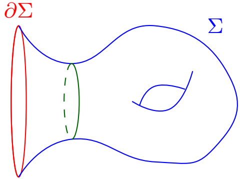
spectral theory

spectrum of laplacian Δ



geodesics

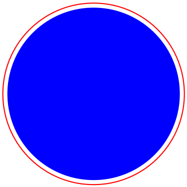


$$\zeta_{\mathrm{Ruelle}}(s) = \prod_{\text{geodesics}} (1 - e^{-s(\text{lengths})})$$

$$\zeta_R(s) \sim s^{\dim H^1(\Sigma,\partial\Sigma)}.\text{torsion} + \text{h.o.t.}$$

error correcting codes

error correcting codes



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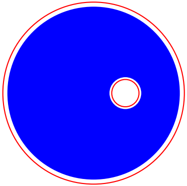

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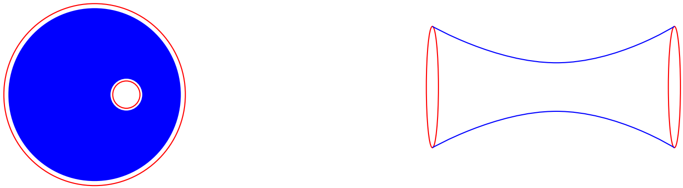

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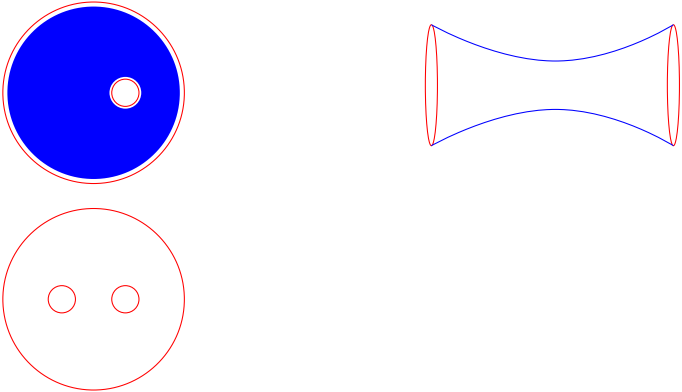

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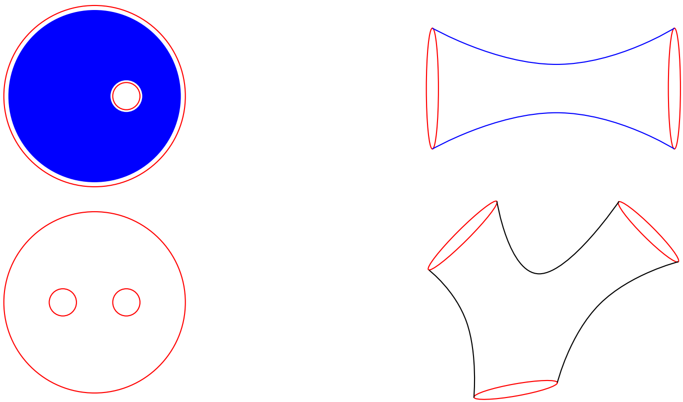

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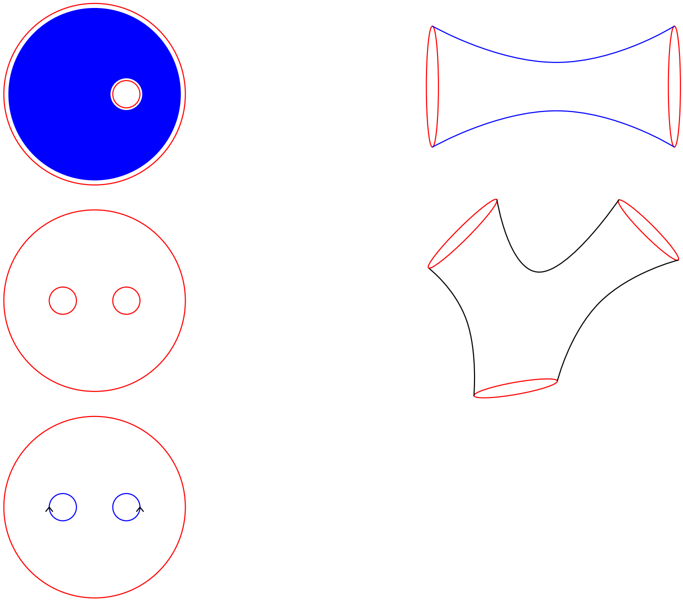

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