Lecture 4 surface patch compilation

What we think we know:

- individual surface patch for 1 logical qubit
- could correct many errors
- logical X, Z gates
  - two other neat gates:
    - H (sort of)
    - CNOT (impractical but transversal)
  - phase gate to get full Clifford group. requires state distillation.
- movement of surface patches on our $\mathbb{Z}_2$ grid of qubits.

What I would love to finish with

- CNOT via lattice surgery
- $|T\rangle$ via teleportation
- distillation of $|T\rangle$ (but this is very murky for me)

A strange implementation of CNOT

- use a mediator qubit
- measure weight-two Paulis.

$|m\rangle$ •   • $|t\rangle$

$|c\rangle$ •

0) initialise $|m\rangle = |+\rangle$

Protocol:
1) M_{ZZI}
2) if -1 evalue then apply IXI
3) M_{IXX}
4) M_{IZI}
5) if -1 evalue then apply IIX

0) $|c\rangle|m\rangle = \left(\alpha|0\rangle + \beta|1\rangle\right) \otimes |+\rangle$

$= \alpha\langle ZI, IX\rangle + \beta\langle -ZI, IX\rangle$

1) $M_{ZZ} \longrightarrow \alpha\langle ZI, \pm ZZ\rangle + \beta\langle -ZI, \pm ZZ\rangle$

$= \alpha\langle ZI, \pm IZ\rangle + \beta\langle -ZI, \mp IZ\rangle$

2) maybe $_{IX} \longrightarrow \alpha\langle ZI, IZ\rangle + \beta\langle -ZI, -IZ\rangle \quad = \alpha|00\rangle + \beta|11\rangle$

3) $|cm\rangle|t\rangle = |cm\rangle \otimes \left(\alpha'\langle Z\rangle + \beta'\langle -Z\rangle\right)$

$$
\begin{aligned}
= \; & \alpha\alpha' \langle ZII, IZI, IIZ\rangle \\
& \alpha\beta' \langle ZII, IZI, -IIZ\rangle \\
& \beta\alpha' \langle -ZII, -IZI, IIZ\rangle \\
& \beta\beta' \langle -ZII, -IZI, -IIZ\rangle
\end{aligned}
$$

$$
\begin{aligned}
M_{IXX} \longrightarrow \; & \alpha\alpha' \langle \quad , IZZ, \pm IXX\rangle \\
& \qquad\quad , -IZZ, \\
& \qquad\quad , -IZZ, \\
& \beta\beta' \langle \quad , IZZ, \pm IXX\rangle
\end{aligned}
$$

4) $M_{IZI} \longrightarrow \langle \pm Z\rangle_m \otimes$

$$
\begin{aligned}
& \alpha\alpha' \langle ZI, \pm IZ\rangle_{ct} \\
& \alpha\beta' \langle ZI, \mp IZ\rangle_{ct} \\
& \beta\alpha' \langle -ZI, \mp IZ\rangle_{ct} \\
& \beta\beta' \langle -ZI, \pm IZ\rangle_{ct}
\end{aligned}
$$

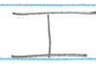
5) maybe X on target $\longrightarrow$

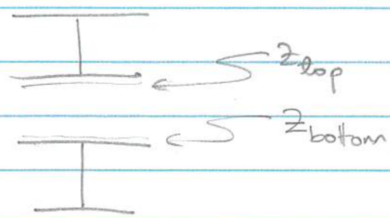
$$
\left.
\begin{aligned}
& \alpha\alpha' \langle ZI, IZ\rangle \\
& \alpha\beta' \langle ZI, -IZ\rangle \\
& \beta\alpha' \langle -ZI, -IZ\rangle \\
& \beta\beta' \langle -ZI, +IZ\rangle
\end{aligned}
\right\} = \alpha|0\rangle|t\rangle + \beta|1\rangle \otimes X|t\rangle
$$

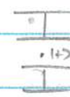
How would I do this using surface patches?

steps 0,2,4,5 are easy, we've seen those ideas last lecture.
steps 1,3?

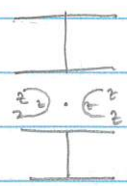
M_{IXX} will be similar to M_{ZZI} so let's focus on measuring ZZ between two surface patches.

- one surface patch  had Z eigenvalue decided by parity of horizontal Z's

- 

- place intermediary qubit in $|+\rangle$ state 

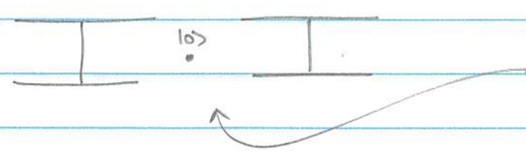
and measure newly created Z plaquettes (X plaquettes will all be +1)

- 

$$M_{plaq\ left} = \begin{smallmatrix} Z & & I \\ & Z & \\ Z & & I \end{smallmatrix} \qquad M_{plaq\ right} = \begin{smallmatrix} I & & Z \\ & Z & \\ I & & Z \end{smallmatrix}$$

$$M_{p\ left} \times M_{p\ right} = \begin{smallmatrix} Z & & Z \\ & I & \\ Z & & Z \end{smallmatrix} = \begin{smallmatrix} Z_{top} \\ Z_{bottom} \end{smallmatrix} = M_{ZZ}$$

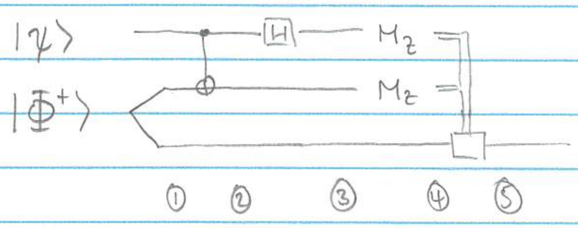
- parity of plaquettes along smooth boundaries equals ZZ evalue!

M_{XX} : 

Measure star ops down rough border.

One final loose end: Magic states

Remember how teleportation worked?



① $\frac{1}{\sqrt{2}}\Big(\alpha\,(000 + 011) + \beta\,(100 + 111)\Big)$

② $\frac{1}{\sqrt{2}}\Big(\alpha\,(000 + 011) + \beta\,(110 + 101)\Big)$

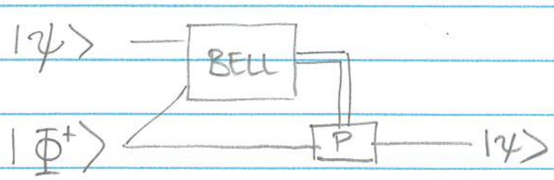
③ $\frac{1}{2}\Big(\alpha\,(000 + 100 + 011 + 111) + \beta\,(010 - 110 + 001 - 101)\Big)$

④

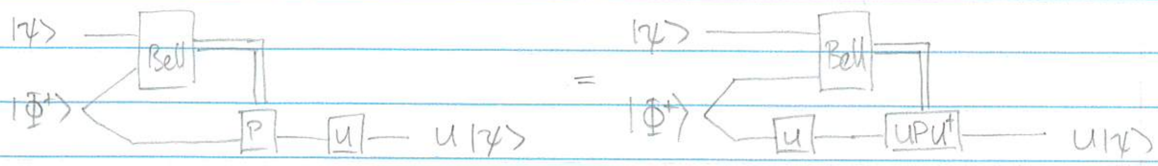
M_{ZII}	M_{IZI}			
0	0	$\alpha\|0\rangle + \beta\|1\rangle$	apply	I
0	1	$\alpha\|1\rangle + \beta\|0\rangle$		X
1	0	$\alpha\|0\rangle - \beta\|1\rangle$		Z
1	1	$\alpha\|1\rangle - \beta\|0\rangle$		X then Z

5 if square box is $Z^{b_1} \cdot X^{b_2}$ then outcome in $|\psi\rangle$

Let's observe that $Z^{b_1} X^{b_2}$ is a Pauli operation and black box this entire observation:

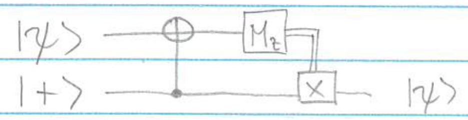


This becomes useful if I want to apply a unitary to $|\psi\rangle$.
I can commute it through the P pauli:



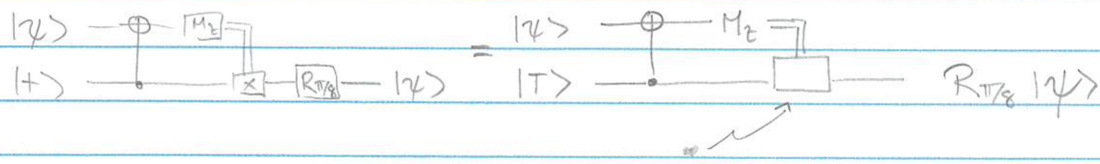
And this becomes really interesting if $UPU^\dagger$ can be applied transversally.
Perhaps it is a Clifford gate. Eg $U = T =$ "$\pi/8$"

We can make this much more relevant to QC by using 1-qubit teleportation:



(X applied if $M_z =$ "-1")

I would like to commute a 'U' gate past $\boxed{X}$ and then past $\oplus$. If U is in $C^2 \sim 2^{nd}$ Clifford hierarchy, passing by $\boxed{X}$ gives a Clifford :) And if U is diagonal, then it will commute past $\oplus$. Therefore, use $\pi/8$ gate!!!



where $|T\rangle = R_{\pi/8}|+\rangle$ $\qquad$ $R_{\pi/8} X R_{-\pi/8}$ ~ proportional to $X \cdot S$.

how would you do this in the surface patch?