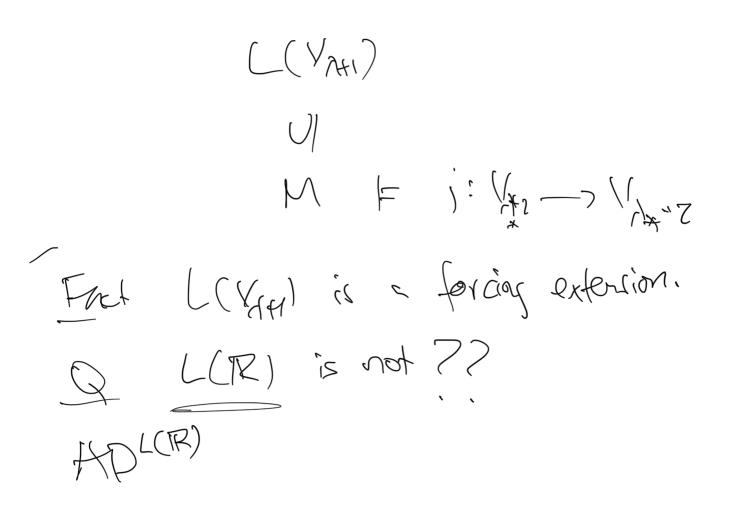
Measurable cardinals and choiceless axioms

Éz is equiremation w/ 16 for all TEVAN master $Gder : (\chi_{H}, T) \longrightarrow (V_{H}, T)$ (Kunon) There is no j: V/HZ -> V/HZ (Schlutenberg) The following are exiconsidert: () ZECA Lo € ZF + J-DC + (j: VATZ) $): \left(\left(\right)^{+1} \right) \longrightarrow \left(\left(\right)^{+1} \right)$ curt(i) < A $\left\lfloor \left(\begin{array}{c} V_{\lambda+1} \end{array} \right) \left\lceil \left[\begin{array}{c} V_{\lambda+1} \end{array} \right] \right\rceil \right\rangle \left[\begin{array}{c} V_{\lambda+1} \end{array} \right]$

 \sim / \cup / \cup / \cup / \cup /

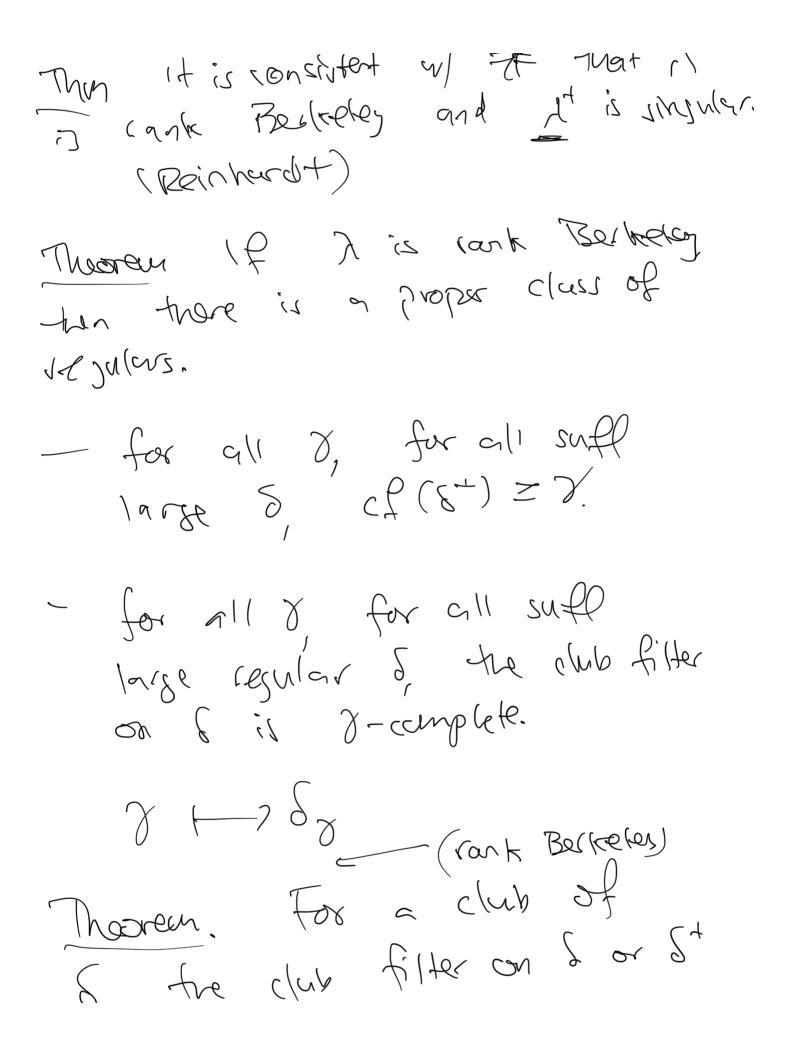


Jensen: Exactly one of the following tokes: () V is close to 2: for all singular th A is singular in L and A^{NL} = A^N () V is for from L: every cordinal is inaccessible in L. Woodin 18 there is an extendible F

and holds OV is close to HOD: for all SWONKI YZK Lite day t A is stagular in (toD and regular dZK, (2) V is four from HOD: for all Sis neasurable in HOD. w-strongly measurable $J: HOD(V_{H}) \longrightarrow HOD(V_{H})$ HOD conjecture. Large cardinals imply V is close to HOD. Mooreon (woodin) & the HOD conjecture is a Reinwordt cordinal true, then - proper class of extendidos is ia consistent.

Mark Reakeley cordinals.

ILV.IL V Def.) is rank Berkeley of for all $|\alpha < \lambda \in B$, the is $j : \sqrt{\beta} - \frac{1}{\beta}$ w with j > q (and critici) < λ). Open is a Reinhardt equiconsistent w/ a rank Berkeley? Theorem (Cutolo), 12 à is any singular RAA Bockeley limit of aterdibles, then 2t is measurable. Gitika: it is consistent u/ ZF that every uncountable cardinal is singular. Q (FISPECE) IS it conjuted w/ 2 beipp sound Berkeley that every condition avove 2 is singular? -2 h ((



is J-complete. IS Fis a filter a set ASX is an itom of F.P. ZSNA: SEFZ i an altrafilter. (AD) F= c(ub filter on Wz $gq = w_2$: $cf(q) = w_1^2$ is an atom $\left\{ d \in \omega_{1} : c f(\alpha) = \omega_{1} \right\}$ is an atoms A filter is atomic if every positive set contains on atom. Mearen 15 thore is a can't Berkeley, Do large regular cardine's S,

the club filter on Sis arbmic. than for suff i-s. Kor. If there's alt Berkeley, then there is a club of 8 s.t. Ja or St is measurable. Mearvable conditions Theorem. (rk Beskeley) For a chib class of cardinals & every S-complete filter on an ordinal extends to La Sconplete y thafilter. Maaren (Kunen) Under AD + DC, Rilly Wi-complete filler on a<0 athors to an concomplete ultradillor.

Ketoren order. If v is an ordinal and visa set, $B_{\nu}(x) = \nu \cdot \operatorname{complete} u \cdot fs \text{ on } X$, ketonen order is an order on u.Ps on ordiacly. Fix an ordinal S. A function f: P(8) -> P(8) is Ketoren if 6 5 is Lipschitz if x 5 and x 5 f(x) ng depends only in xna. (2) If $W \in B_{w_1}(S)$, then $f^{-2}[W] \in B_{w_1}(S)$. xeU q->f(x)eW $\mathcal{C} \mathcal{B}^{(k)}(\mathcal{E})$

Ketonen reducibility: $U \leq_{lk} W$ $rf \quad \exists (Cetemen \quad f: P(\delta) \longrightarrow P(f))$ $S.E- f^{-T} [W] = U.$ Provubly wellfounded (DC) Ultraponer Axian 2 => for clid, (ZF() Mor 7M, Bu (8) is linearly anded by Ketoner veducibility Open Does AD mply theoring of Refored reducibility. Semi-theorit Rank Beckeley continue imply semilihoanty of the later order. Theorem. If is rank Berkeley, N tor mail 8

then for some ?? the Ketoney order on Br(6) ulmost (near: every level has cordiantity = to and every set of in comparables has size = 2.

'Pseudo lesse condina's Kis (1) 00) - Superconepact Def if for all LZK, there is and $\pi: V_{\overline{A}} \longrightarrow V_{A}$ $\lambda < \kappa$ $\mathcal{T}(\mathcal{V}).$ s.E.t is almost supercompact if it is (y w)-supercompact for all V - 19. RD

This IP the read (1, io)-super compact I the least rank Berketley they if a supercompact Fact. If shall is a reak Berkeley, that there is a proper class of almost superconpacts. Thoorem. (Wellordored collector) if & is almost superion part, then for all y < h, if < Ax : x - y > is a sequence of nonempty sets, they those is a set & st. 50 Az 7 fs for all deg and 5 is the sonijedue knose of Vix.

. I wait Superconpart

Or. If or is crown and YZM, then I' than cofile-lites at (east st. Proof. Suppose not. Then take your and (9; = 3 < 97 conversions to yt. By wellbredored collection those is a set I that is the swjectice image of Va and for each & cy, those is a wellorder of x in a of ordertype az. $Sup(q_g) = sup { Vank(g) : f \in G }$ $f: \gamma \times \varsigma \longrightarrow \gamma^{+}$ $f(q, \forall) = rank_{g}(q)$ [f[Exjx0] <5

 $ran(f) = J^{\dagger}$ Proof of wellordored collection formes. Suppose for all B=9, the wellowerd collection lemma holds. Fix <Aq: \$ 297. For each B=9, lef $B_{B} = 79$: $Aom(G) = V_{T} J$ ran (G) nA_{S} for all 3<B Now let j: V_ -> V/ be exemptory A = 19 and j(n) = 19. - Consider Att ran(j) is corrinal in 90 for cofinally damy Bcg, thee's some (ge Bp 1 ran (j)) (if Bfran (j))

 $ron(j) \leq (f_{\lambda} < V_{K})$ Let $S = Sran(f) : f \in BB \cap ran(j)$ For any BEJan(j), for all SEB, on Az is nonempty Defre h: $V_{\underline{x}} \times (\underline{y}) \longrightarrow 5$ h(x,f) = j(f)(x)