Tentative Syllabus

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1 Model theory in set theory
   • Elementary substructures
   • Ultrapowers
   • Stationary sets

2 Inner models
   • The constructible hierarchy
   • Absoluteness and the principle $V = L$
   • The consistency of the Axiom of Choice
   • Condensation and the Generalized Continuum Hypothesis
   • The diamond principle and Suslin’s problem
   • Ordinal definability
   • Descriptive set theory and absoluteness

3 Forcing
   • Partial orders, filters, ultrafilters
   • Names and the forcing theorem
   • Preservation of ZFC and chain conditions
   • The independence of the Continuum Hypothesis
   • The independence of the Axiom of Choice
   • Forcing over $V$
   • Complete Boolean algebras and Vopenka’s Theorem
   • Product forcing and Easton’s Theorem
   • Iterated forcing, Martin’s Axiom, and the Suslin problem
4 Large cardinals

- Inaccessible cardinals
- The Lévy collapse and the Solovay model
- Measurable cardinals, ultrapowers, and Scott’s theorem
- The Lévy-Solovay theorem
- $0^\#$ and Jensen’s covering lemma
- Determinacy and regularity properties
- The Wadge order
- Analytic determinacy from a measurable cardinal
- Measurable cardinals from determinacy
- The inner model $L[U]$
- Strongly compact cardinals and the Singular Cardinals Hypothesis
- Supercompact cardinals and canonical inner models