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Math 54 Second Midterm Fall 2009 Instructor: D.-V. Voiculescu  
 This is a "closed book" exam, so you may not bring in or use notes or the textbook. Calculators are not allowed.

Please write your name, SID and Discussion Section # on everything you hand in, including this sheet of paper on which you have to provide answers to Problem II (the true or false questions). For Problem I you must show the method and calculations you use to get the answers (write the solutions to the questions in Problem I in your blue book). The Requirement is 20 points.

Problem I (3+4+4 pts) Consider the following two ordered bases in  $\mathbb{R}^4$

$$B = \{(0,0,1,0), (0,1,1,0), (1,1,1,0), (1,1,1,1)\} \{b_1, b_2, b_3, b_4\}$$

$$C = \{(0,0,0,1), (1,0,0,1), (0,1,1,0), (1,1,0,1)\} \{c_1, c_2, c_3, c_4\}$$

- Find the transition matrix from C to B.
- Find the orthonormal basis  $D = \{x, y, z, t\}$  which is obtained from C by the Gram-Schmidt procedure.
- Find the 4x4 matrix T with eigenvectors  $x, y, z, t$  for the respective eigenvalues 1, -1, 2, 0.

Problem II (9 pts, each question 1 pt). Check True or False.

	True	False
a) All solutions of $y'' + y = 0$ are bounded.	<input checked="" type="checkbox"/>	<input type="checkbox"/>
b) $y'' - 3y' + 5y = 0$ has two linearly independent solutions $y_1(t), y_2(t)$ on $\mathbb{R}$ with Wronskian at zero $W[y_1, y_2](0) = 3$ .	<input checked="" type="checkbox"/>	<input type="checkbox"/>
c) The function $y(t)$ which is $= e^t$ if $t > 0$ and $= e^{2t}$ if $t \leq 0$ , is a solution of $y'' - 3y' + 2y = 0$ on $\mathbb{R}$ .	<input type="checkbox"/>	<input checked="" type="checkbox"/>
d) $e^{(1+i)\pi}$ is a real number.	<input checked="" type="checkbox"/>	<input type="checkbox"/>
e) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$ is a diagonalizable matrix	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>
f) The matrix $\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ is diagonalizable	<input type="checkbox"/>	<input checked="" type="checkbox"/>
g) The transpose of a symmetric matrix is equal to its inverse	<input type="checkbox"/>	<input checked="" type="checkbox"/>
h) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 2 \end{pmatrix}$ is an orthogonal matrix. <small>Not normalized!</small>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
i) If A is an orthogonal matrix then $A^{-1}$ is always invertible	<input checked="" type="checkbox"/>	<input type="checkbox"/>

$A^t = A^{-1}$  yes.

orthogonal matrix true.

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