Math 54 First Midterm Fall 2010 Instructor: D.-V. Voiculescu

This is a "closed book" exam, so you may not bring in or use notes or the textbook. Calculators are not allowed.

Please write your name, SID and Discussion Section # on everything you hand in,including this sheet of paper on which you have to provide answers to Problem II (the true or false questions). For Problem I you must show the method and calculations you use to get the answers (write the solutions to the questions in Problem I in your blue book).The Requirement is 20 points.

Problem I (4 + 3 + 3 + 4 pts). Throughout this problem we assume $\begin{pmatrix}a\\b\end{pmatrix}, \begin{pmatrix}c\\d\end{pmatrix}$ is a basis of R^2.

a) Prove that $B = \{X,Y,Z,T\}$ where $X=\begin{pmatrix}a\\b\\0\\0\end{pmatrix}, Y=\begin{pmatrix}c\\d\\0\\0\end{pmatrix}, Z=\begin{pmatrix}0\\0\\a\\b\end{pmatrix}, T=\begin{pmatrix}0\\0\\c\\d\end{pmatrix}$ is a basis of R^4.

b) Find the coordinate vectors of the vectors of the basis B with respect to the basis $C = \{M,N,P,Q\}$ where $M=\begin{pmatrix}0\\1\\0\\0\end{pmatrix}, N=\begin{pmatrix}0\\1\\1\\0\end{pmatrix}, P=\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, Q=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$.

c) Find a basis of Nul A, where A is the 4x4 matrix [X|Y|N|N].

d) Compute det[M|X|N|T] .

Problem II (6 pts, each question 1 pt). Check True or False.

	True	False
a) In a vector space V if $u,v \in V$ and $u > v > 0$ then $u+v > 2v$.	X	
b) If U, V, W are vector spaces and U is a subspace of V and V is a subspace of W, then U is a subspace of W.	X	
c) If $T : R^2 \to R^3$ is a linear map, then $T\begin{pmatrix}0\\1\end{pmatrix}$ and $T\begin{pmatrix}1\\0\end{pmatrix}$ are always linearly independent.		X
d) If U, V are subspaces of a vector space W then so is their intersection.		X
e) If A, B , C are 3x3 matrices then $(A(B+C))^T = B^TA^T + C^TA^T$.	X	
f) A system of 3 linear equations with 9 unknowns may have no solution.	X	