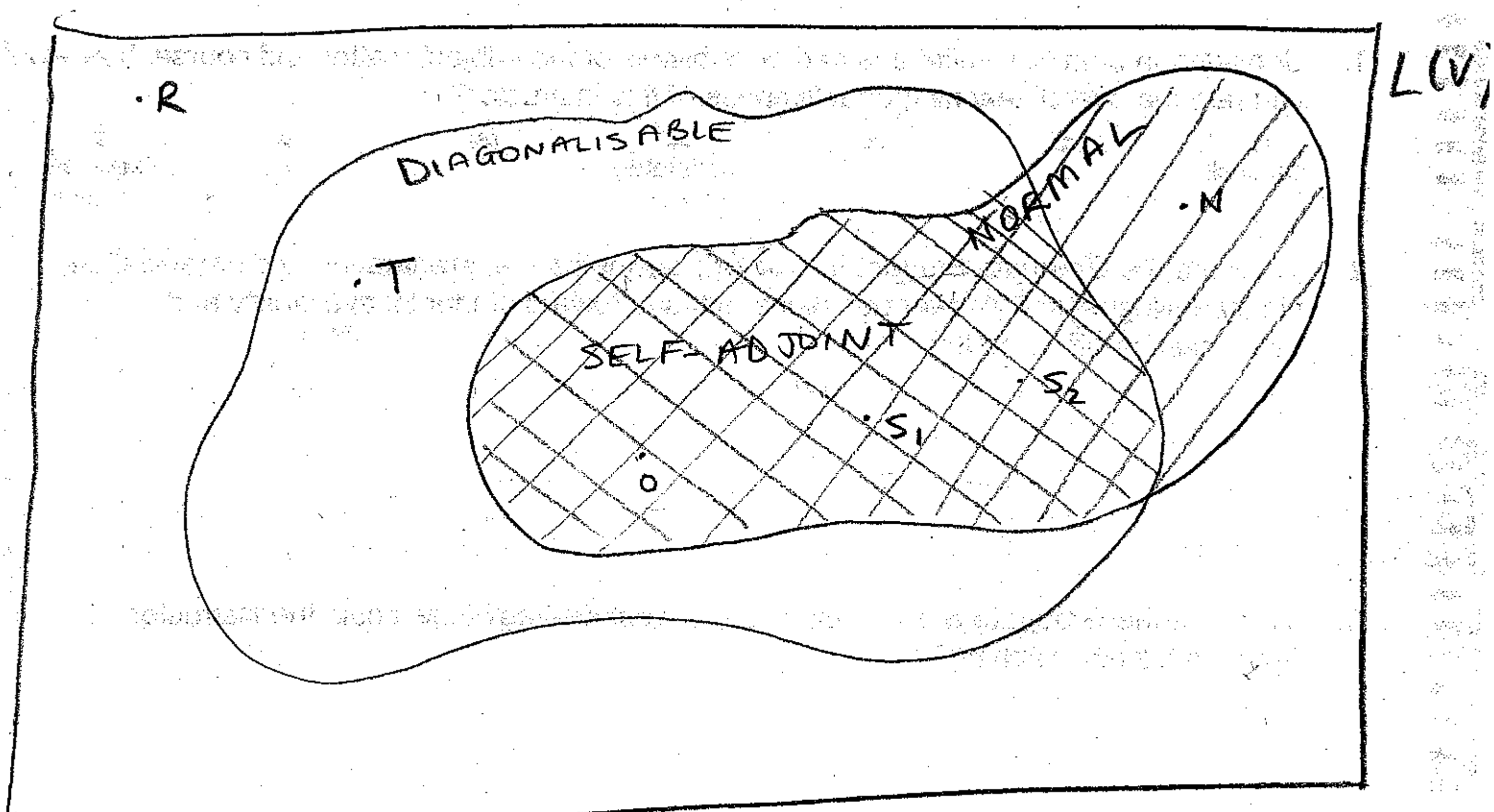
LET V BE FINITE DIMENSIONAL <u>REAL</u> VECTOR SPACE ;
$\langle , \rangle$ AN INNER PRODUCT ON V.

<u>eg</u>
- $V = \mathbb{R}^n$, $\langle x, y \rangle = x_1 y_1 + \cdots + x_n y_n$
- $V = P_{\leq n}(\mathbb{R})$, $\langle p, q \rangle = \int_0^1 p(x) q(x)\, dx$

THE VECTOR SPACE OF OPERATORS ON , $L(V)$, "LOOKS LIKE"



$$S_1 : \mathbb{R}^n \longrightarrow \mathbb{R}^n$$
$$\underline{x} \longmapsto \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix} \underline{x}$$

$$S_2 : \mathbb{R}^n \longrightarrow \mathbb{R}^n , \quad A \in \mathrm{Mat}_n(\mathbb{R})$$
$$\underline{x} \longmapsto A\underline{x} \quad \& \quad A = A^t .$$

$$N : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$\underline{x} \longmapsto \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \underline{x}$$

$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ DEFINED ON BASIS $\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix} \right)$ BY $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$T\left(\begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix} \right) = - \begin{bmatrix} 1 \\ \sqrt{2} \end{bmatrix}$$

$$R : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$$
$$\underline{x} \longmapsto \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

ALL THESE EXS ARE USING $\langle , \rangle$ DEFINED ABOVE !

KNOW WHY THESE OPERATORS ARE (NON-) EXAMPLES ! ASK ME IF YOU HAVE ANY QUESTIONS