Math 110, Fall 2015.

Homework 9, optionally due Oct 28.

Prob 1. Show that the function taking the pair $((x_1, x_2, x_3), (y_1, y_2, y_3)) \in \mathbb{R}^3 \times \mathbb{R}^3$ to $x_1y_2 + x_2y_3 + x_3y_1$ is not an inner product.

Prob 2. Use the dot product to show that the diagonals of a rhombus are perpendicular to each other.

Prob 3. Prove that, for all complex numbers a_j, b_j, $j = 1, \dots, n$,

$$\Big|\sum_{j=1}^{n} a_j\overline{b_j}\Big|^2 \leq \left(\sum_{j=1}^{n} j\,|a_j|^2\right)\left(\sum_{j=1}^{n} \frac{|b_j|^2}{j}\right).$$

Prob 4. Let $e_1, \dots, e_m$ be an orthonormal list of vectors. Prove that $v \in \operatorname{span}(e_1, \dots, e_m)$ if and only if

$$\|v\|^2 = \sum_{j=1}^{m} |\langle v, e_j\rangle|^2.$$

Prob 5. Consider the space $\mathcal{P}_3(\mathbb{R})$ with the inner product

$$\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)dx.$$

Use the Gram-Schmidt algorithm to orthonormalize the basis $1, x, x^2, x^3$.

Prob 6. Find $p \in \mathcal{P}_3(\mathbb{R})$ such that $p(0) = 0$, $p'(0) = 0$, and

$$\int_{0}^{1} |1 + 4x - p(x)|^2 dx$$

is as small as possible.