Math 110, Fall 2015.

Homework 8, due Oct 21.

Prob 1. Let v be an eigenvector of $T \in \mathcal{L}(V)$ with eigenvalue λ. Show that

$$(T^3 + T^2 - 2T + I)v = (\lambda^3 + \lambda^2 - 2\lambda + 1)v.$$

How does this observation generalize?

Prob 2. Let $S, T \in \mathcal{L}(V)$ and suppose S is invertible. Prove that, for any polynomial $p \in \mathcal{P}(\mathbb{F})$,

$$p(STS^{-1}) = S\,p(T)\,S^{-1}.$$

Prob 3. Give an example of an operator $T \in \mathcal{L}(P_2(\mathbb{C}))$ with eigenvalues 3 and 4 which is not diagonalizable.

Prob 4. Let V be a finite-dimensional real vector space and let $T \in \mathcal{L}(V)$. Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(\lambda) := \dim \operatorname{range}(T - \lambda I).$$

Which condition on T is equivalent to f being a continuous function?

Prob 5. Suppose V is a finite-dimensional complex vector space, $T \in \mathcal{L}(V)$ is diagonalizable, and all eigenvalues of T are strictly below 1 in absolute value. Given $\phi \in V'$ and $v \in V$, what is $\lim_{n\to\infty} \phi(T^n v)$?

Prob 6. Explain how diagonalizability of (square) matrices you learned in Math 54 is related to the diagonalizability of operators.