Math 110, Fall 2015.

Homework 7, due Oct 14.

Prob 1. Prove that every polynomial of odd degree with real coefficients has a real zero.

Prob 2. Let $p \in \mathcal{P}_n(\mathbb{C})$ for some n and suppose there exist distinct real numbers x_0, x_1, …, x_n such that $p(x_j) \in \mathbb{R}$ for all $j = 0, \ldots, n$. Prove that all coefficients of p are real.

Prob 3. [Lagrange interpolation.] Prove *using linear algebra*: given distinct *data sites* x_j and arbitrary *data* y_j, $j = 0, \ldots, n$, there is a unique polynomial $p \in \mathcal{P}_n(\mathbb{R})$ such that $p(x_j) = y_j$, for all $j = 0, \ldots, n$.

Prob 4. Give an example of a linear map $T : \mathcal{P}_3(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R})$ with an invariant subspace other than $\{0\}$, the whole space $\mathcal{P}_3(\mathbb{R})$, or $\operatorname{null} T$.

Prob 5. Given S, $T \in \mathcal{L}(V)$ such that $ST = TS$, prove that $\operatorname{null} S$ and $\operatorname{range} S$ are invariant under T.

Prob 6. Let $T : \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R}) : p \mapsto p - p'$. Find all eigenvalues and eigenvectors of T.