Math 110, Fall 2015.

Homework 5, due Sept 30.

Prob 1. Suppose V and W are finite-dimensional vector spaces. Let $v \in V$, and consider

$$E := \{T \in \mathcal{L}(V, W) : Tv = 0\}.$$

(a) Show that E is a subspace of $\mathcal{L}(V, W)$.

(b) Suppose $v \neq 0$. What is $\dim E$?

Prob 2. Suppose u, w are vectors in V and U, W are subspaces of V such that $u + U = w + W$. Does this imply that $U = W$?

Prob 3. Let U be a subspace of V such that V/U is finite-dimensional. Prove or disprove: V is isomorphic to $U \times (V/U)$.

Prob 4. Let U be a subspace of V, and consider $T \in \mathcal{L}(V, W)$. Let π denote the quotient map from V onto V/U. Prove that $U \subset \operatorname{null} T$ if and only if there exists $S \in \mathcal{L}(V/U, W)$ such that $T = S \circ \pi$.