**Math 110, Fall 2015.**

# Homework 2, due Sept 9.

**Prob 1.** Prove or give a counterexample: if $U_1$, $U_2$, $W$ are subspaces of $V$ such that

$$U_1 + W = U_2 + W,$$

then $U_1 = U_2$.

**Prob 2.** Suppose

$$U = \{(x, y, x + y, x - y, 2x) \in \mathbb{F}^5 : x, y \in \mathbb{F}\}.$$

Find three subspaces $W_1$, $W_2$, $W_3$ of $\mathbb{F}^5$, none of which equals $\{0\}$, such that $\mathbb{F}^5 = U \oplus W_1 \oplus W_2 \oplus W_3$.

**Prob 3.** Suppose that the list $v_1, v_2, v_3, v_4$ is linearly independent in $V$. Show that the list $v_1 - v_2, v_1 + v_2, v_3 - v_2, v_4 - v_1$ is also linearly independent in $V$.

**Prob 4.** Does the statement of Problem 3 still hold if we replace 'linearly independent' by 'a basis'?

**Prob 5.** Prove that the space $\mathbb{R}^R$ is infinite-dimensional.

**Prob 6.** What is the dimension of

- (a) $\mathbb{C}$ as a vector space over $\mathbb{C}$?
- (b) $\mathbb{C}$ as a vector space over $\mathbb{R}$?
- (c) $\mathbb{C}^2$ as a vector space over $\mathbb{C}$?
- (d) $\mathbb{C}^2$ as a vector space over $\mathbb{R}$?