Math 110, Fall 2015.

Homework 14, due Dec 2.

Prob 1. Let V be a complex finite-dimensional vector space and let $T \in \mathcal{L}(V)$ have eigenvalues -1, 0, 1. Given the dimensions of the corresponding nullspaces below, determine the Jordan normal form of T.

λ	$\dim \operatorname{Null}(T - \lambda \mathbb{I})$	$\dim \operatorname{Null}(T - \lambda \mathbb{I})^2$	$\dim \operatorname{Null}(T - \lambda \mathbb{I})^3$	$\dim \operatorname{Null}(T - \lambda \mathbb{I})^4$	$\dim \operatorname{Null}(T - \lambda \mathbb{I})^5$
-1	3	5	6	6	6
0	2	4	6	7	7
1	3	4	5	5	5

Prob 2. Let $T \in \mathcal{L}(\mathcal{P}_3(\mathbb{C}))$ be the operator

$$T : f(x) \mapsto f(x-1) + x^3 f'''(x)/3.$$

Find the Jordan normal form and a Jordan basis for T.

Prob 3. Let V be a real vector space. Prove that $T \in \mathcal{L}(V)$ is invertible if and only if $T_{\mathbb{C}}$ is invertible.

Prob 4. Suppose V is a real vector space of dimension 8. Let $T \in \mathcal{L}(V)$ be such that $T^2 + T + \mathbb{I}$ is nilpotent. Prove that $(T^2 + T + \mathbb{I})^4 = 0$. Give an example of such an operator T.

Prob 5. Let V be a real vector space of dimension n and let $T \in \mathcal{L}(V)$ be such that $\operatorname{Null} T^{n-3} \neq \operatorname{Null} T^{n-2}$. How many eigenvalues of $T_{\mathbb{C}}$ can be nonreal?

Prob 6. Let V be a real finite-dimensional space and let $T \in \mathcal{L}(V)$. Prove that $T_{\mathbb{C}}$ has only real eigenvalues if and only if there exists a basis of V consisting of generalized eigenvectors of T.