**Math 110, Fall 2015.**

# Homework 13, due Nov 25.

**Prob 1.** Let $V$ be a complex $n$-dimensional space and let $T \in \mathcal{L}(V)$ be such that $\operatorname{null} T^{n-3} \neq \operatorname{null} T^{n-2}$. How many distinct eigenvalues can $T$ have?

**Prob 2.** Let $V = \mathcal{P}_3(\mathbb{C})$ and let $D \in \mathcal{L}(V)$ be the differentiation operator. Find a square root of $\mathbb{I} + D$.

**Prob 3.** Let $V$ be a complex (finite-dimensional) vector space and let $T \in \mathcal{L}(V)$. Prove that there exist operators $D$ and $N$ in $\mathcal{L}(V)$ such that $T = D + N$, $D$ is diagonalizable, $N$ is nilpotent, and $DN = ND$.

**Prob 4.** Suppose that $V$ is a complex vector space of dimension $n$. Let $T \in \mathcal{L}(V)$ be invertible. Let $p$ denote the characteristic polynomial of $T$ and let $q$ denote the characteristic polynomial of $T^{-1}$. Prove that

$$q(z) = \frac{z^n}{p(0)}\, p\Big(\frac{1}{z}\Big) \qquad \text{for all} \quad z \in \mathbb{C} \setminus \{0\}.$$

**Prob 5.** Suppose the Jordan form of an operator $T \in \mathcal{L}(V)$ consists of Jordan blocks of sizes $3 \times 3$, $4 \times 4$, $1 \times 1$, $5 \times 5$, $2 \times 2$, corresponding to eigenvalues $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_2$, $\lambda_1$, respectively. Assuming that $\lambda_i \neq \lambda_j$ for $i \neq j$, find the minimal and the characteristic polynomial of $T$.