Math 110, Fall 2015.

Homework 12, due Nov 18.

Prob 1. Prove that dim Range T equals the number of nonzero singular values of T.

Prob 2. Let $S \in \mathcal{L}(V)$. (a) Prove that S is an isometry if and only if all the singular values of S equal 1. (b) Give a nontrivial example of an isometry in your favorite vector space.

Prob 3. Suppose $T \in \mathcal{L}(V)$, m is a positive integer, and $v \in V$ is such that $T^{m-1}v \neq 0$ but $T^m v = 0$. Prove that

$$v, Tv, T^2v, \ldots, T^{m-1}v$$

is linearly independent.

Prob 4. Show that the only linear operator on a (finite-dimensional) inner product space that is both normal and nilpotent is the zero operator.

Prob 5. Suppose V is a complex (finite-dimensional) vector space and $T \in \mathcal{L}(V)$. Prove that T is nilpotent if and only if 0 is the only eigenvalue of T,