Math 110, Fall 2015.

Homework 1, due Sept 2, 2015.

Problem 1. Suppose that $\{0, 1, x\}$ is a field with exactly three elements. What do the addition and multiplication table *have to be* in that case?

Based on the addition and multiplication tables you get, check that it is indeed a field. What is the natural way to think of this field (and x)?

Problem 2. Let $n \in \mathbb{N}$. Is $\mathbb{Z}^n$ a vector space over $\mathbb{Z}$? Over $\mathbb{Q}$? Over $\mathbb{R}$? Explain your answers.

Problem 3. Suppose $a \in \mathbb{F}$ (field), $v \in V$ (vector space over $\mathbb{F}$), and $av = 0$. Prove that $a = 0$ or $v = 0$.

Problem 4. Prove that any field $\mathbb{F}$ is also a vector space over itself, with the field addition used as vector addition and the field multiplication used as scalar multiplication.

Problem 5. Prove that the set of all differential real-valued functions f on the interval $(0, 3)$ such that $f'(1) = a$ is a subspace of $\mathbb{R}^{(0,3)}$ if and only if $a = 0$.