MATH 110 , FALL 2013

LET V BE FINITE DIMENSIONAL <u>COMPLEX</u> VECTOR SPACE ;

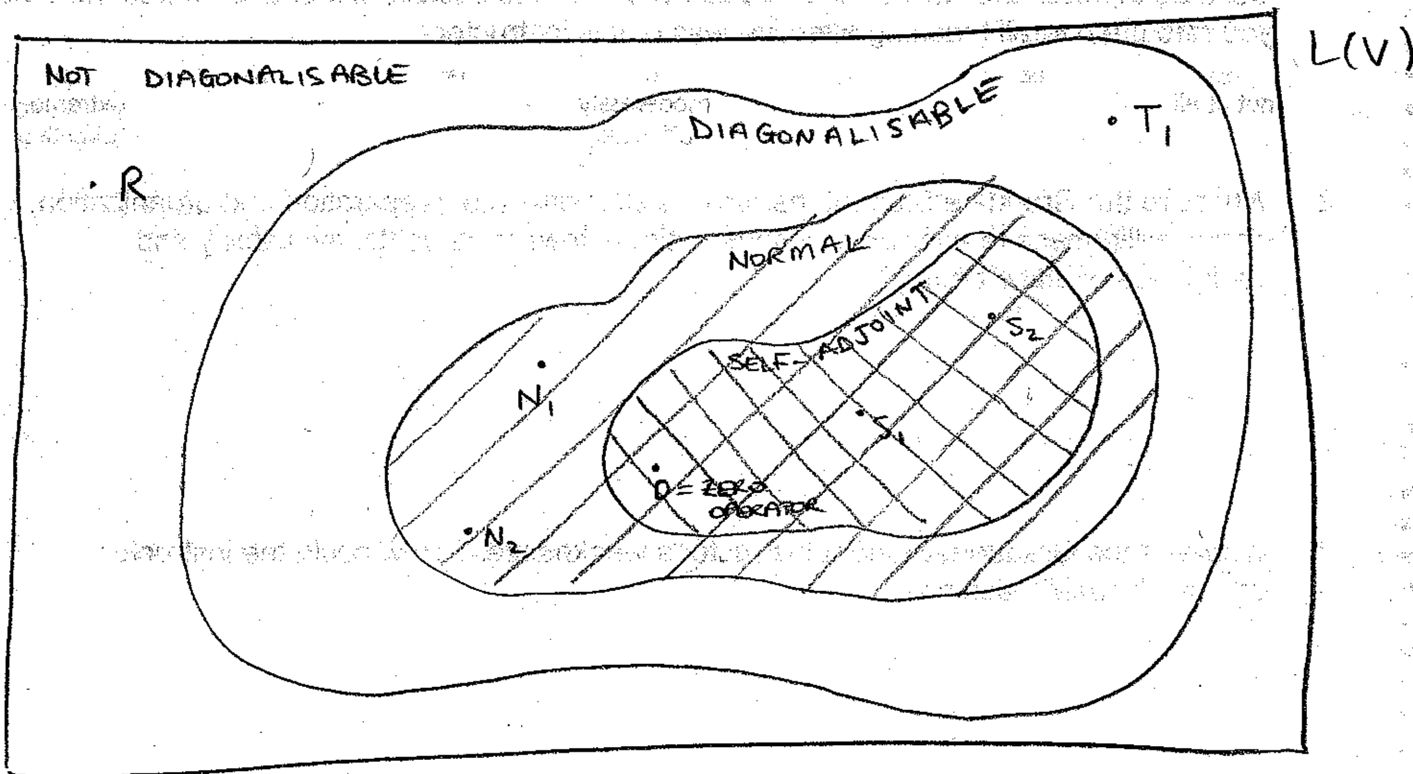
$\langle , \rangle$ AN INNER PRODUCT ON V.

<u>eg</u>: • $V = \mathbb{C}^n$, $\langle x, y \rangle = x_1\overline{y_1} + \cdots + x_n\overline{y_n}$

• $V = P_{\leq n}(\mathbb{C})$, $\langle p, q \rangle = \int_0^1 p(x)\overline{q}(x)\,dx$,

WHERE , IF $q(x) = a_0 + \cdots + a_n x^n$, $\overline{q}(x) = \overline{a_0} + \cdots + \overline{a_n} x^n$.

THE VECTOR SPACE OF OPERATORS ON V, $L(V)$, "LOOKS LIKE"



$S_1 : \mathbb{C}^n \longrightarrow \mathbb{C}^n$

$\underline{x} \longmapsto \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix} \underline{x}$, $a_1, \ldots, a_n \in \mathbb{R}$

"SELF-ADJ $\iff$ NORMAL + $\mathbb{R}$-EVALS"

$S_2 : \mathbb{C}^n \longrightarrow \mathbb{C}^n$, $A \in Mat_n(\mathbb{C})$

$\underline{x} \longmapsto A\underline{x}$ & $A = \overline{A}^t$

ALL THESE EXS ARE USING $\langle , \rangle$ DEFINED ABOVE !

$N_1 : \mathbb{C}^n \longrightarrow \mathbb{C}^n$

$\underline{x} \longmapsto \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix} \underline{x}$, <u>SOME</u> $a_i \notin \mathbb{R}$

$N_2 : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$

$\underline{x} \longmapsto \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \underline{x}$

$T_1 : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$, DEFINED ON BASIS $\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ \sqrt{-1} \end{bmatrix} \right)$ BY $T_1\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = 1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$T_1\left(\begin{bmatrix} 1 \\ \sqrt{-1} \end{bmatrix}\right) = 2 \cdot \begin{bmatrix} 1 \\ \sqrt{-1} \end{bmatrix}$

$R : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$

$\underline{x} \longmapsto \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \underline{x}$.

KNOW WHY THESE OPERATORS ARE (NON-) EXAMPLES! ASK ME IF YOU HAVE ANY QUESTIONS