Worksheet 12/2. Math 110, Fall 2015.

These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework. Send me an email if you have any questions!

Finding Jordan form/basis

Throughout this worksheet V will always be a finite dimensional vector space over $F = \mathbb{C}$.

1. Let $T \in L(V)$, where $\dim V = 5$. Suppose that T admits exactly two distinct eigenvalues λ, μ. What are the allowed Jordan forms:

a) if $\dim(T - \lambda I) = 1, \dim(T - \mu I) = 1$?

b) if $\dim(T - \lambda I) = 2, \dim(T - \mu I) = 2$?

c) if $\dim(T - \lambda I) = 2, \dim(T - \mu I) = 1$?

2.

a) Give an example of an operator $T \in L(\mathbb{C}^6)$ with precisely three 2-Jordan blocks, one (-1)-Jordan block, and such that $\text{null}(T-2)^2 = 4$.

b) Can there exist an operator $T \in L(\mathbb{C}^6)$ with precisely three 2-Jordan blocks, one (-1)-Jordan block, and such that $\text{null}(T-2)^2 = 5$?

3. Determine the Jordan form of the following operators $T \in L(V)$.

a) $T : \mathbb{C}^3 \to \mathbb{C}^3 \; ; \; x \mapsto \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} x$

b) $T : \mathbb{C}^4 \to \mathbb{C}^4 \; ; \; x \mapsto \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} x$

(*You may use without proof that $\lambda = 1, -1$ are the only eigenvalues of T.*)

4. Find Jordan bases for the operators in Problem 3.