Quiz 9. Discussion Section 106. Math 110 Fall 2014.

Name: Solution

1. Let $(V, \langle, \rangle)$ be a Hermitian space, $T : V \to V$ an anti-Hermitian operator on V. Suppose that $v \in V$ is an eigenvector of T with eigenvalue λ. Prove that $U = \{u \in V \mid \langle u, v\rangle = 0\}$ has the following property: if $u \in U$ then $T(u) \in U$.

   Solution: let $u \in U$, so that $\langle u, v\rangle = 0{=}0$. To show that $T(u) \in U$ we must show that $\langle T(u), v\rangle = 0$: now, we have

   $$\langle T(u), v\rangle = \langle u, T^{+}(v)\rangle = \langle u, -T(v)\rangle = \langle u, -\lambda v\rangle = -\lambda\langle u, v\rangle = 0,$$

   where we have used that $T = -T^{+}$, since T is assumed anti-Hermitian.