

Quiz 7. Discussion Section 106. Math 110 Fall 2014.

Name: Solution

1. Determine the inertia indices (p, q) of the following quadratic form

$$Q = -x_1^2 + 2x_1x_2 - x_2x_3 + 2x_3^2.$$

Solution: Completing the square gives

$$Q = -(x_1 - x_2)^2 + (x_2 - \frac{1}{2}x_3)^2 + \frac{7}{4}x_3^2$$

and letting

$$u_1 = x_1 - x_2, \quad u_2 = x_2 - \frac{1}{2}x_3, \quad u_3 = \frac{\sqrt{7}}{2}x_3,$$

gives

$$Q = -u_1^2 + u_2^2 + u_3^2.$$

Thus, we have

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & \sqrt{7}/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Since this last matrix is invertible we find that $(p, q) = (2, 1)$.