Quiz 7. Discussion Section 103. Math 110 Fall 2014.

Name: Solution

1. Determine the inertia indices (p, q) of the following quadratic form

$$Q = 2x_1^2 + 2x_1x_3 - x_2x_3 + x_2^2.$$

Solution: Completing the square gives

$$Q = 2(x_1 + \frac{1}{2}x_3)^2 + (x_2 - \frac{1}{2}x_3)^2 - \frac{3}{4}x_3^2$$

So that if we set

$$u_1 = \sqrt{2}(x_1 + \frac{1}{2}x_3), \quad u_2 = x_2 - \frac{1}{2}x_3, \quad u_3 = \frac{\sqrt{3}}{2}x_3,$$

then

$$Q = u_1^2 + u_2^2 - u_3^2.$$

Moreover, the above defines a change of coordinates since

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & -1/2 \\ 0 & 0 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

and the given matrix is invertible. Hence, $(p, q) = (2, 1)$.