Quiz 6. Discussion Section 106. Math 110 Fall 2014.

Name: Solution

1. Consider the following matrix
$$A = \begin{bmatrix} 2 & -2 \\ 1 & 3 \end{bmatrix}.$$

   Determine a lower triangular invertible matrix L, an upper triangular invertible matrix U and a permutations matrix P such that $A = LPU$.

   Solution: Row-reduce A using only row scalings and addition of rows to rows below (no swaps; only 'downward row-reduction') to obtain

$$A \sim X = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = L^{-1}A.$$

   Column-reduce X using only column scalings and addition of columns to proceeding columns (no swaps; only 'rightward column-reduction') to obtain

$$X \sim P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = XU^{-1}.$$

   Then, $L = AX^{-1}$, $U = XP^{-1} = X$, so that

$$L = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$