Quiz 6. Discussion Section 103. Math 110 Fall 2014.

Name: Solution

1. Consider the following matrix

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}.$$

   Determine a lower triangular invertible matrix L, an upper triangular invertible matrix U and a permutation matrix P such that $A = LPU$.

   Solution: Row-reduce A using only row scalings and addition of rows to rows below (so no swaps; only 'downward row-reduction') to obtain

   $$A \sim X = \begin{bmatrix} 1 & -2/3 \\ 0 & 1 \end{bmatrix} = L^{-1}A$$

   Then, column-reduce X using only row scalings and addition of columns to proceeding columns to obtain

   $$X \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = P = XU^{-1}$$

   Hence, $L = AX^{-1}$ and $U = P^{-1}X = X$. Thus,

   $$L = \begin{bmatrix} 3 & 0 \\ 1 & 8/3 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & -2/3 \\ 0 & 1 \end{bmatrix}$$