Quiz 5. Discussion Section 106. Math 110 Fall 2014.

Name: Solution

1. Consider the vectors

$$x = \begin{bmatrix} -1 \\ 2 \\ -3 \\ 4 \end{bmatrix}, \quad y = \begin{bmatrix} -4 \\ 3 \\ -2 \\ 1 \end{bmatrix}.$$

Let $A = xy^t$, a 4×4 matrix (!): compute the adjoint $\text{adj}(A)$.

Solution: The matrix is

$$A = \begin{bmatrix} 4 & -3 & 2 & -1 \\ -8 & 6 & -4 & 2 \\ 12 & -9 & 6 & -3 \\ -16 & 12 & -8 & 4 \end{bmatrix}$$

The adjoint matrix $\text{adj}(A)$ is defined to be $\text{adj}(A) = [c_{ij}]$, where $c_{ij} = A_{ji}$ is the ji^{th} cofactor. As the rows/columns of A are all multiples of the a single row/column (eg, the first row/last column) then the same is true for *any* 3×3 submatrix of A. Hence, any cofactor admits linearly dependent columns, so that $A_{ji} = 0$, for every j, i. Thus, $\text{adj}(A) = 0$ is the zero matrix.