Quiz 5. Discussion Section 106. Math 110 Fall 2014.

Name: Solution

1. Consider the vectors

\[ x = \begin{bmatrix} -1 \\ 2 \\ -3 \\ 4 \end{bmatrix}, \quad y = \begin{bmatrix} -4 \\ 3 \\ -2 \\ 1 \end{bmatrix}. \]

Let \( A = xy^\top \), a \( 4 \times 4 \) matrix (!): compute the adjoint \( \text{adj}(A) \).

Solution: The matrix is

\[
A = \begin{bmatrix}
4 & -3 & 2 & -1 \\
-8 & 6 & -4 & 2 \\
12 & -9 & 6 & -3 \\
-16 & 12 & -8 & 4
\end{bmatrix}
\]

The adjoint matrix \( \text{adj}(A) \) is defined to be \( \text{adj}(A) = [c_{ij}] \), where \( c_{ij} = A_{ji} \) is the \( ji^{th} \) cofactor. As the rows/columns of \( A \) are all multiples of the a single row/column (eg, the first row/last column) then the same is true for any \( 3 \times 3 \) submatrix of \( A \). Hence, any cofactor admits linearly dependent columns, so that \( A_{ji} = 0 \), for every \( j, i \). Thus, \( \text{adj}(A) = 0 \) is the zero matrix.