Quiz 4. Discussion Section 106. Math 110 Fall 2014.

Name: Solution

1. Consider the quadratic form

$$Q(x,y) = -x^2 + 8xy + 3y^2.$$

Determine a 2×2 matrix $A = [a_{ij}]$ and a symmetric bilinear form

$$B : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}\ ;\ (\underline{u}, \underline{v}) \mapsto \sum_{i,j=1}^{2} u_i a_{ij} v_j,$$

such that $Q(x,y) = B((x,y),(x,y))$.

Solution: We need $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ such that

$$Q(x,y) = B((x,y),(x,y)) = a_{11}x^2 + (a_{12}+a_{21})xy + a_{22}y^2 \implies a_{11} = -1,\ a_{22} = 3,\ a_{12}+a_{21} = 8.$$

We must also have $B(\underline{u}, \underline{v}) = B(\underline{v}, \underline{u})$ so that $a_{12} = a_{21}$. Hence, We find that $2a_{12} = 8 \implies a_{12} = a_{21} = 4$. Thus, the matrix is

$$A = \begin{bmatrix} -1 & 4 \\ 4 & 3 \end{bmatrix},$$

and

$$B(\underline{u}, \underline{v}) = -u_1v_1 + 4u_1v_2 + 4u_2v_1 + 3u_2v_2.$$

You can also use the formula

$$B(\underline{u}, \underline{v}) = \frac{1}{2}\left(Q(u_1+v_1, u_2+v_2) - Q(u_1,u_2) - Q(v_1,v_2)\right).$$