

Quiz 4. Discussion Section 103. Math 110 Fall 2014.

Name: Solution

1. Consider the quadratic form

$$Q(x, y) = x^2 + 6xy + 2y^2.$$

Determine a 2×2 matrix $A = [a_{ij}]$ and a symmetric bilinear form

$$B : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R} ; (\underline{u}, \underline{v}) \mapsto \sum_{i,j=1}^2 u_i a_{ij} v_j,$$

such that $Q(x, y) = B((x, y), (x, y))$.

Solution: We need $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ such that

$$Q(x, y) = B((x, y), (x, y)) = a_{11}x^2 + (a_{12} + a_{21})xy + a_{22}y^2 \implies a_{11} = 1, a_{22} = 2, a_{12} + a_{21} = 6.$$

We must also have $B(\underline{u}, \underline{v}) = B(\underline{v}, \underline{u})$ so that $a_{12} = a_{21}$. Hence, We find that $2a_{12} = 6 \implies a_{12} = a_{21} = 3$. Thus, the matrix is

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix},$$

and

$$B(\underline{u}, \underline{v}) = u_1 v_1 + 3u_1 v_2 + 3u_2 v_1 + 2u_2 v_2.$$

You can also use the formula

$$B(\underline{u}, \underline{v}) = \frac{1}{2} (Q(u_1 + v_1, u_2 + v_2) - Q(u_1, u_2) - Q(v_1, v_2)).$$