Quiz 3. Discussion Section 106. Math 110 Fall 2014.

Name: Solution

1. Let $M = \{2 \times 2 \text{ matrices with } \mathbb{R} \text{ entries}\}$, a vector space over $\mathbb{R}$, and let $A = \{C \in M \mid C^t = -C\} \subset M$ be the subspace of *antisymmetric matrices*. Here we have used the notion of the transpose of a matrix:

$$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \implies C^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}.$$

Find a basis B of A, taking care to prove that B is indeed a basis.

Solution: The given condition $C^t = -C$ implies that $C \in A$ precisely when

$$C = \begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix}.$$

Hence, we have that the set $\left\{ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$ is a basis, since this set spans A and the (only) vector is nonzero, hence linearly independent.