Quiz 3. Discussion Section 103. Math 110 Fall 2014.

Name: Solution

1. Let $M = \{2 \times 2$ matrices with $\mathbb{R}$ entries$\}$, a vector space over $\mathbb{R}$, and let $S = \{A \in M \mid A^t = A\} \subset M$ be the subspace of *symmetric matrices*. Here we have used the notion of the transpose of a matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \implies A^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}.$$

Find a basis B of S, taking care to prove that B is indeed a basis.

Solution: Using the condition $A^t = A$, we see that $A \in M$ precisely when

$$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix} = a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + d \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Now, the set $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ is linearly independent: if there are $c_1, c_2, c_3 \in \mathbb{R}$ such that

$$c_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ c_2 & c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

then we must have $c_1 = c_2 = c_3 = 0$. Hence, the given set is a basis.