

Quiz 1. Discussion Section 106. Math 110 Fall 2014.

Name: Solution

1. Express all of the roots of the equation $w^2 + 2w + 1 - \frac{\sqrt{-1}}{4} = 0$ in the form $a + b\sqrt{-1}$.

Solution: We use the quadratic formula to obtain the roots

$$w = \frac{1}{2} \left(-2 \pm \sqrt{4 - 4(1 - \sqrt{-1}/4)} \right) = \frac{1}{2} \left(-2 \pm \sqrt{\sqrt{-1}} \right)$$

Hence, we need to find a square root of $a = \sqrt{-1}$: write $a = e^{\sqrt{-1}\pi/2}$. Then, a square root $b = re^{\sqrt{-1}\theta}$ ($r \geq 0$, $\theta \in [0, 2\pi)$) of a satisfies $b^2 = a$, so that

$$r^2 = 1, \quad 2\theta = \pi/2.$$

Hence, we can take $r = 1$, $\theta = \pi/4$, so that $b = e^{\sqrt{-1}\pi/4} = \frac{1}{\sqrt{2}} + \frac{\sqrt{-1}}{\sqrt{2}}$. Hence, the roots are

$$w = -1 \pm \frac{1}{\sqrt{2}} \left(1 + \sqrt{-1} \right)$$
$$\implies w \in \left\{ (-1 + 1/2\sqrt{2}) + \sqrt{-1}/2\sqrt{2}, (-1 - 1/2\sqrt{2}) - \sqrt{-1}/2\sqrt{2} \right\}.$$