

Quiz 1. Discussion Section 103. Math 110 Fall 2014.

Name: Solution

1. Express all of the roots of the equation $w^2 - \sqrt{-4}w - (1 + \frac{\sqrt{-1}}{4}) = 0$ in the form $a + b\sqrt{-1}$.

Solution: We use that quadratic formula to obtain the roots

$$w = \frac{1}{2} \left(\sqrt{-4} + \sqrt{-4 + 4(1 + \frac{\sqrt{-1}}{4})} \right) = \frac{1}{2} \left(\sqrt{-4} + \sqrt{\sqrt{-1}} \right).$$

We need to obtain a square root of the complex number $a = \sqrt{-1}$: in polar form we can write $a = e^{\sqrt{-1}\pi/2}$, where $r \geq 0, \theta \in [0, 2\pi)$. A square root $b = re^{\sqrt{-1}\theta}$ of a must satisfy $b^2 = a$. Hence, we have

$$r^2 = 1, \quad 2\theta = \pi/2,$$

and we can take $b = e^{\sqrt{-1}\pi/4} = \frac{1}{\sqrt{2}} + \frac{\sqrt{-1}}{\sqrt{2}}$. Hence, we have

$$w \in \left\{ \frac{1}{2\sqrt{2}} + \sqrt{-1}\left(1 + \frac{1}{2\sqrt{2}}\right), -\frac{1}{2\sqrt{2}} + \sqrt{-1}\left(1 - \frac{1}{2\sqrt{2}}\right) \right\}$$