Worksheet August 28th. Math 110 Fall2014.

These problems are intended as supplementary material to the homework exercises and will hopefully give you some more practice with actual examples. In particular, they may be easier/harder than homework. Problems with an asterisk (*) should be more challenging than the rest.

1. Find the inverse z^{-1} of the following complex numbers

   (a) $z = 2 + \sqrt{-1}$

   (b) $z = 3 - 2\sqrt{-1}$

   (c) $z = \sqrt{\sqrt{-1}}$

2. Find all of the solutions to $x^6 - 1 = 0$.

3. Determine if the following subsets are nonempty (ie, are there any complex numbers z satisfying the given constraints); if so, describe all such complex numbers.

   (a) $\{z \in \mathbb{C} \mid |z - 1| = |z + 1|\}$

   (b) $\{z \in \mathbb{C} \mid |z - 2| \leq 1\} \cap \{z \in \mathbb{C} \mid |z| \leq 1\}$

   (c) $\{z \in \mathbb{Z} \mid |z - 2| \geq |z - 1|\}$

4. Let $\mu_n = \{z \in \mathbb{C} \mid z^n = 1\} = \{z_1, \ldots, z_n\}$. Show that $\sum_{1 \leq i < j \leq n} z_i z_j = 0$. Can you generalise this to a relation involving products $z_i z_j z_k$, with i, j, k distinct?