1. Find the sum of the series

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n\cdot(n+1)}+\cdots$$

Determine whether the series $\sum a_n$ converges, where

2. $a_n=\sqrt{n+1}-\sqrt{n}$,

3. $a_n=\dfrac{\sqrt{n+1}-\sqrt{n}}{n}$.

Prove that the following series diverge:

4. $1+\dfrac{1}{2}-\dfrac{1}{3}+\dfrac{1}{4}+\dfrac{1}{5}-\dfrac{1}{6}+\cdots$

5. $\dfrac{1}{\sqrt{1\cdot 2}}+\dfrac{1}{\sqrt{2\cdot 3}}+\dfrac{1}{\sqrt{3\cdot 4}}+\cdot$

For each of the following seies, find the interval of convergence and examine convergence at the endpoints of the interval:

6. $\displaystyle\sum_{n\geq 0}\frac{(n!)^2}{(2n)!}x^n$

7. $\displaystyle\sum_{n\geq 0}\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n$ (α is a given real number)

8. $\displaystyle\sum_{n>0}n^n x^{n^2}$

9. $\displaystyle\sum_{n\geq 0}\left(\frac{x}{\cos n}\right)^n$

10. Compute coefficients of the powers series

$$(1+x+x^2+\cdots+x^n+\cdots)^3.$$