

Practice problems from past exams

1. Does there exist a function f whose partial derivatives are:

(a) $f_x = y \sin xy, \quad f_y = x \sin xy?$

(b) $f_x = x \sin xy, \quad f_y = y \sin xy?$

2. The graph of a function $z = f(x, y)$ is the surface of revolution obtained by rotating the parabola $z = x^2/2$, lying in the plane $y = 0$, about the z -axis. Compute the directional derivative of this function at the point $(x, y) = (10, 25)$ in the direction of the vector $(0, -1)$.

3. Express the derivative $\partial^2 f / \partial u \partial v$ of the composite function obtained from a twice differentiable function $f(x, y)$ by the substitution $x = u + v, y = uv$.

4. Give an example of a function f with $f_{xy} \neq f_{yx}$.

5. Find all differentiable functions $f(x, y)$ such that $f_x = y \exp xy$.

6. Compute the gradient of the function $1/|\vec{r}|$.

7. Find all values and partial derivatives at the point $(a, b) = (5, 4)$ of the function $x(a, b)$ defined implicitly by the equation $x^4 - ax^2 + b = 0$.

8. Find and classify all critical points of the function $f = xy - x^3/3 - y^3/3$.

9. Find the global maximum and minimum values of the function $\sqrt{x^2 + y^2}$ in the region

$$-1 \leq x \leq 3, \quad -2 \leq y \leq 4.$$

10. Find all critical points and critical values of the function $x + y + z$ under the constraint $2xy^2z^3 = 1$.

The angular momentum vector \vec{M} of a moving particle with respect to a center is defined as $\vec{M} = m(\dot{\vec{r}} \times \vec{r})$, where \vec{r} is the radius-vector of the particle with respect to the center, $\dot{\vec{r}}$ is the velocity vector, and m is the mass of the particle.

11. Derive the angular momentum conservation law: If a particle moves according to Newton's 2nd law " $F = ma$ " in any force field directed to a center, then the angular momentum vector with respect to this center does not change.

12. Astronomers observe a strange planet, Swingus, moving around the Sun $(0, 0, 0)$ according to the law: $x = a \cos t, y = b \sin t, z = 0$. Describe the orbit of Swingus, and find out if the motion obeys the angular momentum conservation law.