Answers to HW3

3, Section 5. The required identity about codimensions (which is the minimal number of linear equations needed to describe a given subspace) expresses the property of the three groups of equations for V_1, V_2, V_3 to be indepedent altogether. The transversality of V_i to $V_j \cap V_k$ means that the equations of V_i are independent from the rest. But any dependency between the equations of V_j and V_k would mean that V_j is not transverse to V_k, and hence not transverse to $V_k \cap V_i$.

8, Section 5. It is good to know the authors finally realized that $x^2 + y^2 - z^2 = 1$ describes not a paraboloid, but a hyperboloid, namely hyporboloid of revolution of one sheet. The sphere $x^2 + y^2 + z^2 = a$ of radius $\sqrt{a}$ does not meet the hyperboloid as long as $a > 0$ remains < 1. At $a = 1$, it touches the hyperbolod along the circle $x^2 + y^2 = 1, z = 0$, and for $a > 1$ intersects transversally along two parallel circles in the planes $z = \pm\sqrt{a-1}$.

10, Section 5. In local coordinates, the equation $f(x) = x$ of fixed points (which therefore form a closed set) can be rewritten as $g(x) := f(x) - x = 0$. The Lefschetz condition means that at a fixed point x_0, $(dg)_{x_0}$ is non-degenerate, and hence, by the Inverse Function Theorem, the fixed point is isolated. It remains to notice that in a compact space a closed set with no limit points must be finite.

7, Section 6. In $\mathbf{C}^n$, multiplication by unitary scalars $z \mapsto e^{\pi i t} z$ preserves the unit sphere $|z_1|^2 + \cdots + |z_n|^2 = 1$ of dimension $2n-1$, and provides a homotopy between the identity (at $t = 0$) and the antipodal map (at $t = 1$).

9, Section 6. At $t = 0$, we have the identity function x, which is a diffeomorphism from $\mathbb{R} \to \mathbb{R}$ transverse to $Z = \{0\}$. For $t > 0$, the function is still x withinn the interval $|x| < 1/t$, but is identically 0 outside the interval $|x| < 2/t$ (and hence does not fit any of the requisite classes). As $t \to 0$, the interval $|x| < 1/t$ expands to the entire line, while the "bad" region runs away "to infinity and beyond".