Math 215B. Spring 2020. Final exam

`Rules:` *Below you'll find 7 problems, of which I ask you to solve 4. If you submit more than 4 solutions, I will grade them, but will count only 4 best. Your exam papers are due at 4 p.m. on Thursday, May 14. You are* **not** *allowed to consult other students or any outside sources (books, internet). You are allowed, however, to resort to our textbook* Fomenko–Fuchs, *the articles linked from the course website (though you probably won't need them), other materials from the course website (such as homework assignments and answers, and my notes for Lectures 15-29 — which can now be found there), as well as your own lecture notes for Lectures 1-14. Browsing the website of Math 215A from the Fall semester is also OK if you find it useful. Good luck!*

1. For bundles $E \xrightarrow{S^2} S^4$, determine all possible (group and) ring structures of $H^*(E)$. (Don't forget to support your list of possibilities with examples demonstrating that the possibilities are realized indeed.)

2. Compute Stiefel-Whitney, Euler, and Pontryagin characteristic classes of (the tangent bundle of) the grassmannian $G(4,2)$.

3. Show that in a closed n-dimensional manifold, any $\mathbb{Z}_2$-coefficient homology class of degree $< n/2$ can be represented by the fundamental class of a closed submanifold.

4. Prove that Milnor's manifolds (see the 2nd paragraph on p. 594 in Fomenko–Fuchs) form a set of generators in the cobordism ring Ω^O.

5. Verify Atiyah-Segal's completion theorem in the example of $\mathbb{Z}_2$-equivariant K-theory of the point by computing the ring $K^*(K(\mathbb{Z}_2, 1))$ understood as the inverse limit over finite-cell approximations of the space, and comparing it to the appropriate completion of the representation ring $R[\mathbb{Z}_2]$.

6. Prove Harnack's inequality (1876): The number of ovals (= connected components) of a non-singular degree d algebraic curve on the real projective plane does not exceed $(d^2 - 3d + 4)/2$.

7. Prove that the nth Chern class of any complex vector bundle over S^{2n} is divisible by $(n-1)!$.